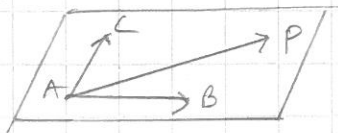


Esim. Pisteet $A = (2, -1, 1)$, $B = (2, 0, 3)$ ja $C = (4, 2, 0)$ määrittävät tason. Onko piste $D = (3, -2, 2)$, $E = (2, 2, 7)$ tasolla? Missä pisteessä taso leikkaa z-akselin?

Ratk.



$$\begin{aligned}
 P \text{ on tasolla} &\Leftrightarrow \vec{AP} = r\vec{AB} + t\vec{AC} \\
 \Leftrightarrow \vec{AP} &= r(\vec{j} + 2\vec{k}) + t(2\vec{i} + 3\vec{j} - \vec{k}) \\
 &= 2t\vec{i} + (r + 3t)\vec{j} + (2r - t)\vec{k} \\
 \Rightarrow P &= (2 + 2t, -1 + r + 3t, 1 + 2r - t)
 \end{aligned}$$

$$D \text{ on tasolla} \Leftrightarrow \begin{cases} 3 = 2 + 2t & (1) \\ -2 = -1 + r + 3t & (2) \\ 2 = 1 + 2r - t & (3) \end{cases}$$

$$(1) : t = \frac{1}{2}$$

$$(2) : -2 = -1 + r + 3 \cdot \frac{1}{2} \Leftrightarrow r = -\frac{5}{2}$$

$$\text{Järk. (3): } 2 = 1 + 2 \cdot \left(-\frac{5}{2}\right) - \frac{1}{2} \Leftrightarrow 2 = 1 - 5 - \frac{1}{2} \quad \downarrow \\
 \Rightarrow \text{yhtälöryhmällä ei ole ratk.} \Rightarrow \underline{D \text{ ei ole tasolla}}$$

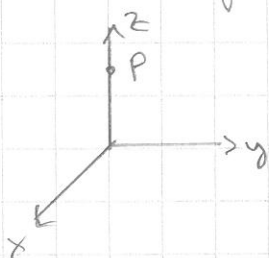
$$E \text{ on tasolla} \Leftrightarrow \begin{cases} 2 = 2 + 2t & (1) \\ 2 = -1 + r + 3t & (2) \\ 7 = 1 + 2r - t & (3) \end{cases}$$

$$(1) : t = 0$$

$$(2) : 2 = -1 + r \Leftrightarrow r = 3$$

$$\text{Järk. (3): } 7 = 1 + 2 \cdot 3 - 0 \quad \checkmark$$

$$\Rightarrow \text{yhtälöryhmällä on ratk.} \Rightarrow \underline{E \text{ on tasolla}}$$



$$P \text{ on z-akselilla} \Leftrightarrow x = y = 0$$

$$\Leftrightarrow \begin{cases} x = 2 + 2t = 0 & \Leftrightarrow t = -1 \\ y = -1 + r + 3t = 0 & \leftarrow \text{inj.} \Rightarrow -1 + r + 3 \cdot (-1) = 0 \\ & \Leftrightarrow r = 4 \end{cases}$$

$$\Rightarrow P = (2 + 2 \cdot (-1), -1 + 4 + 3 \cdot (-1), 1 + 2 \cdot 4 - (-1)) = \underline{(0, 0, 10)}$$

8.4



$$A = (5, 0, -4), B = (-2, 5, -1), P = (1, -2, 5)$$

$$\text{Piste } Q = (x, y, z) \text{ on tasolla} \Leftrightarrow \vec{PQ} \perp \vec{AB}$$

$$\vec{AB} = -7\vec{i} + 5\vec{j} + 3\vec{k}$$

$$\vec{PQ} = (x-1)\vec{i} + (y+2)\vec{j} + (z-5)\vec{k}$$

$$\Rightarrow \vec{AB} \cdot \vec{PQ} = -7(x-1) + 5(y+2) + 3(z-5) = 0$$

$$\Rightarrow \underline{-7x + 5y + 3z + 2 = 0}$$