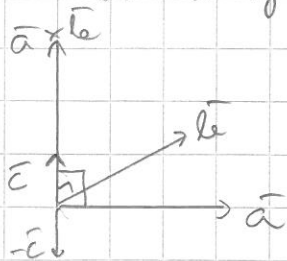


$$4.4 \quad \vec{a} = \vec{i} + \vec{j} + \vec{k}$$

$$\vec{b} = 2\vec{i} - \vec{j} + 3\vec{k}$$



$$a) \quad \vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 1 \\ 2 & -1 & 3 \end{vmatrix} = \begin{vmatrix} 1 & 1 \\ -1 & 3 \end{vmatrix} \vec{i} - \begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} \vec{j} + \begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix} \vec{k}$$

$$= (1 \cdot 3 - 1 \cdot (-1)) \vec{i} - (1 \cdot 3 - 1 \cdot 2) \vec{j} + (1 \cdot (-1) - 1 \cdot 2) \vec{k}$$

$$= \underline{4\vec{i} - \vec{j} - 3\vec{k}}$$

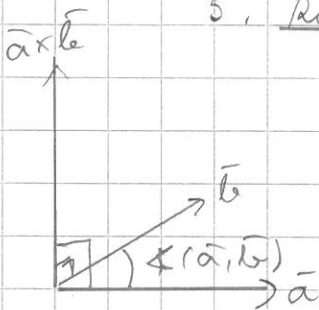
$\vec{a} \cdot (\vec{a} \times \vec{b}) = 1 \cdot 4 + 1 \cdot (-1) + 1 \cdot (-3) = 0 \Rightarrow \vec{a} \perp \vec{a} \times \vec{b} \%$
 $\vec{b} \cdot (\vec{a} \times \vec{b}) = 2 \cdot 4 - 1 \cdot (-1) + 3 \cdot (-3) = 0 \Rightarrow \vec{b} \perp \vec{a} \times \vec{b} \%$

$$b) \quad |\vec{a} \times \vec{b}| = \sqrt{4^2 + (-1)^2 + (-3)^2} = \sqrt{26}$$

$\vec{a} \times \vec{b}$: suuntainen yksikkövektori: $\vec{c} = \frac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|} = \frac{1}{\sqrt{26}} (4\vec{i} - \vec{j} - 3\vec{k})$

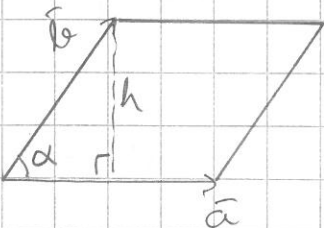
Var. $\pm \vec{c} = \pm \frac{1}{\sqrt{26}} (4\vec{i} - \vec{j} - 3\vec{k})$

5. Pistitöiden suuntavektori



$$|\vec{a} \times \vec{b}| = |\vec{a}| |\vec{b}| \sin(\alpha, \vec{b})$$

Geometrisen tuloksen:

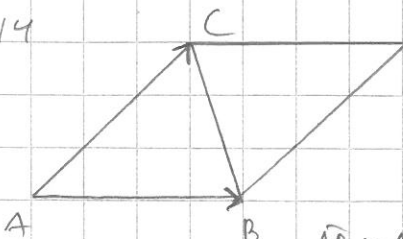


$$\sin \alpha = \frac{h}{|\vec{b}|} \quad | \cdot |\vec{b}| \quad (\Rightarrow) \quad h = |\vec{b}| \sin \alpha$$

$$A = |\vec{a}| h = |\vec{a}| |\vec{b}| \sin \alpha = |\vec{a} \times \vec{b}|$$

$$\boxed{A = |\vec{a} \times \vec{b}|} \quad \text{SUUNNIKKAAN PINTA-ALA}$$

5.14



$$A = (3, -1, 2), \quad B = (1, 2, -2), \quad C = (-1, 3, 2)$$

$$\vec{AB} = -2\vec{i} + 3\vec{j} - 4\vec{k}$$

$$\vec{AC} = -4\vec{i} + 4\vec{j}$$

$$\vec{AB} \times \vec{AC} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -2 & 3 & -4 \\ -4 & 4 & 0 \end{vmatrix} = \begin{vmatrix} 3 & -4 \\ 4 & 0 \end{vmatrix} \vec{i} - \begin{vmatrix} -2 & -4 \\ -4 & 0 \end{vmatrix} \vec{j} + \begin{vmatrix} -2 & 3 \\ -4 & 4 \end{vmatrix} \vec{k}$$

$$= (3 \cdot 0 - (-4) \cdot 4) \vec{i} - (-2 \cdot 0 - (-4) \cdot (-4)) \vec{j} + (-2 \cdot 4 - 3 \cdot (-4)) \vec{k}$$

$$= 16\vec{i} + 16\vec{j} + 4\vec{k}$$

Kolmion ala: $A_B = \frac{1}{2} A_N = \frac{1}{2} \sqrt{16^2 + 16^2 + 4^2} = \frac{1}{2} \sqrt{528} = \frac{1}{2} \sqrt{16 \cdot 33} = \frac{1}{2} \cdot 4 \sqrt{33} = 2\sqrt{33}$