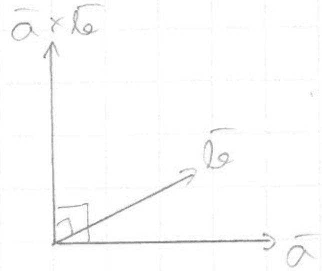
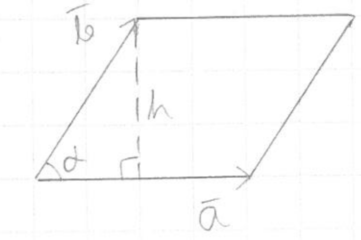
5. Ristitulon sovelluksia



$$|\bar{a}\times\bar{b}| = |\bar{a}|\,|\bar{b}|\sin(\bar{a},\bar{b})$$

Geometrinen tulkinta

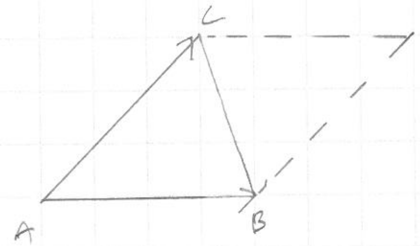


Vektorien $\bar{a}$ ja $\bar{b}$ virittämä suunnikas

$$\sin\alpha = \frac{h}{|\bar{b}|} \quad |\cdot|\bar{b}| \quad \Leftrightarrow \quad h = |\bar{b}|\sin\alpha$$

$$A = |\bar{a}|h = |\bar{a}|\,|\bar{b}|\sin\alpha = |\bar{a}\times\bar{b}|$$

5.14



$A=(3,-1,2)$, $B=(1,2,-2)$, $C=(-1,3,2)$

$$\overline{AB} = -2\bar{i} + 3\bar{j} - 4\bar{k}$$
$$\overline{AC} = -4\bar{i} + 4\bar{j}$$

$$\overline{AB}\times\overline{AC} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ -2 & 3 & -4 \\ -4 & 4 & 0 \end{vmatrix} = \begin{vmatrix} 3 & -4 \\ 4 & 0 \end{vmatrix}\bar{i} - \begin{vmatrix} -2 & -4 \\ -4 & 0 \end{vmatrix}\bar{j} + \begin{vmatrix} -2 & 3 \\ -4 & 4 \end{vmatrix}\bar{k}$$

$$= (3\cdot 0 - (-4)\cdot 4)\bar{i} - (-2\cdot 0 - (-4)\cdot(-4))\bar{j} + (-2\cdot 4 - 3\cdot(-4))\bar{k}$$

$$= 16\bar{i} + 16\bar{j} + 4\bar{k}$$

$$A_s = |\overline{AB}\times\overline{AC}| = \sqrt{16^2+16^2+4^2} = \sqrt{528} = \sqrt{16\cdot 33} = \sqrt{16}\sqrt{33} = 4\sqrt{33}$$

Kolmion pinta-ala: $A_k = \frac{1}{2}A_s = \frac{1}{2}\cdot 4\sqrt{33} = \underline{2\sqrt{33}}$