

3. Pistetulo

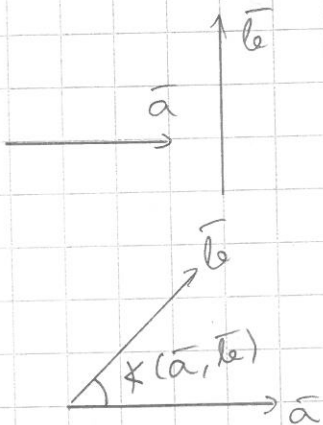
$$\vec{a} = x_1 \vec{i} + y_1 \vec{j} + z_1 \vec{k}, \quad \vec{b} = x_2 \vec{i} + y_2 \vec{j} + z_2 \vec{k}$$

$$\vec{a} \cdot \vec{b} = x_1 x_2 + y_1 y_2 + z_1 z_2 \quad \text{PISTETULO}$$

Huom. 1° $\vec{a} \cdot \vec{b}$ on luku, ei vektori.

2° Pistetulossa piste (\cdot) on aina merkittävä näköyeiin.

3° Pistetulolle pätee "tavalliset" lausekkeet. (n. 29)



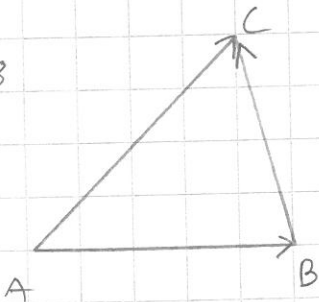
Olkoon $\vec{a}, \vec{b} \neq \vec{0}$

$$\vec{a} \perp \vec{b} \Leftrightarrow \vec{a} \cdot \vec{b} = 0 \quad \text{KOHTISUORUUSEHTO}$$

$$\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos(\vec{a}, \vec{b})$$

$$\Rightarrow \cos(\vec{a}, \vec{b}) = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|}$$

3.8



$$A = (-2, t, 4) \quad B = (3, 0, -1) \quad C = (1, -2, -3)$$

$$\vec{AB} = 5\vec{i} - t\vec{j} - 5\vec{k}$$

$$\vec{AC} = 3\vec{i} + (-2-t)\vec{j} - 7\vec{k}$$

$$\vec{BC} = -2\vec{i} - 2\vec{j} - 2\vec{k}$$

a) $\sphericalangle B = 90^\circ \Leftrightarrow \vec{AB} \perp \vec{BC} \Leftrightarrow \vec{AB} \cdot \vec{BC} = 5(-2) + (-t)(-2) + (-5)(-2) = 0$

$$\Leftrightarrow -10 + 2t + 10 = 0 \quad \Leftrightarrow 2t = 0 \quad \Leftrightarrow t = 0$$

b) $\sphericalangle C = 90^\circ \Leftrightarrow \vec{AC} \perp \vec{BC} \Leftrightarrow \vec{AC} \cdot \vec{BC} = 3(-2) + (-2-t)(-2) + (-7)(-2) = 0$

$$\Leftrightarrow -6 + 4 + 2t + 14 = 0 \quad \Leftrightarrow 2t = -12 \quad \Leftrightarrow t = -6$$

$$|TAI: |AB| = \sqrt{(-2-3)^2 + (t-0)^2 + (4-(-1))^2} = \sqrt{t^2 + 50}$$

$$|AC| = \sqrt{(-2-1)^2 + (t-(-2))^2 + (4-(-3))^2} = \sqrt{(t+2)^2 + 58}$$

$$|BC| = \sqrt{(-3-1)^2 + (0-(-2))^2 + (-1-(-3))^2} = \sqrt{12}$$

a) $\sphericalangle B = 90^\circ$ Pythagoras: $|AC|^2 = |BC|^2 + |AB|^2$

$$\Leftrightarrow (t+2)^2 + 58 = 12 + (t^2 + 50)$$

$$\Leftrightarrow t = 0$$