7.15 $y = f(x) = -x^4 + 4x^2 - 2x$

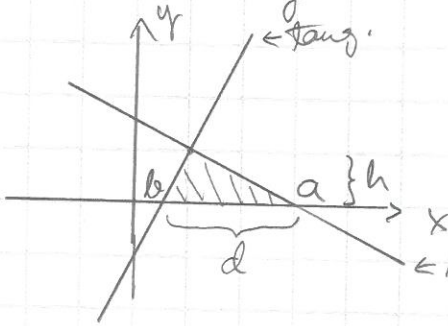
$f(1) = -1^4 + 4\cdot 1^2 - 2\cdot 1 = 1 \Rightarrow$ piste $: (1,1)$

$f'(x) = -4x^3 + 8x - 2 \Rightarrow k_t = f'(1) = -4\cdot 1^3 + 8\cdot 1 - 2 = 2$

tangentti: $y - 1 = 2(x-1) \Leftrightarrow y = 2x - 1$

$k_t \cdot k_n = -1 \Leftrightarrow k_n = -\frac{1}{k_t} = -\frac{1}{2}$

normaali: $y - 1 = -\frac{1}{2}(x-1) \Leftrightarrow y = -\frac{1}{2}x + \frac{3}{2}$

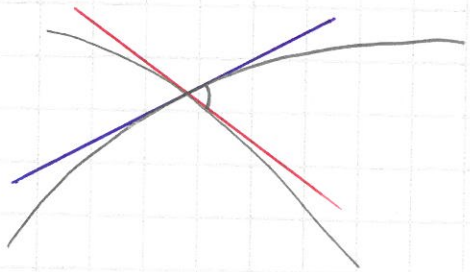


$(a,0): 0 = -\frac{1}{2}a + \frac{3}{2} \Leftrightarrow a = 3$

$(b,0): 0 = 2b - 1 \Leftrightarrow b = \frac{1}{2}$

$A = \frac{1}{2}\cdot d \cdot h = \frac{1}{2}\left(3 - \frac{1}{2}\right)\cdot 1 = \underline{\frac{5}{4}}$

Käyrien leikkauskulma: leikkauspisteeseen piirrettyjen tangenttien välinen kulma



7.13 Leikkauspisteet: $\begin{cases} y = -\frac{1}{2}x^2 + 3 = f(x) \\ y = \frac{1}{4}x^2 + \frac{3}{2}x + 3 = g(x) \end{cases}$

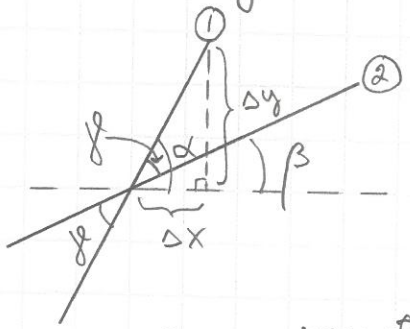
$\Leftrightarrow \frac{1}{4}x^2 + \frac{3}{2}x + 3 = -\frac{1}{2}x^2 + 3 \quad |\cdot 4$

$\Leftrightarrow x^2 + 6x = -2x^2 \quad \Leftrightarrow 3x^2 + 6x = 0 \quad \Leftrightarrow 3x(x+2) = 0$

$\Leftrightarrow x = \begin{cases} 0 \\ -2 \end{cases}$

1° $x = -2 \quad f'(x) = -x \Rightarrow k_{t1} = f'(-2) = -(-2) = 2$

$g'(x) = \frac{1}{2}x + \frac{3}{2} \Rightarrow k_{t2} = g'(-2) = \frac{1}{2}\cdot(-2) + \frac{3}{2} = \frac{1}{2}$

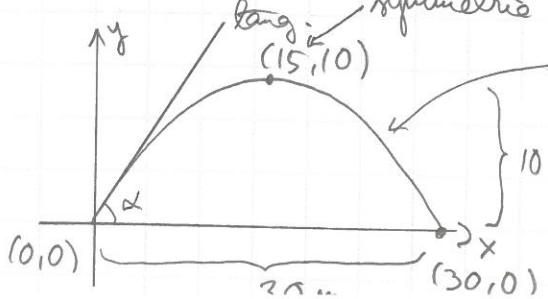


$k_{t1} = \frac{\Delta y}{\Delta x} = \tan\alpha = 2 \Rightarrow \alpha \approx 63{,}43°$

$k_{t2} = \frac{\Delta y}{\Delta x} = \tan\beta = \frac{1}{2} \Rightarrow \beta \approx 26{,}57°$

$\Rightarrow$ leikkauskulma: $\gamma = \alpha - \beta \approx \underline{37°}$

7.19



$y = ax^2 + bx + c$

$\begin{cases} (0,0): a\cdot 0^2 + b\cdot 0 + c = 0 \\ (30,0): a\cdot 30^2 + b\cdot 30 + c = 0 \\ (15,10): a\cdot 15^2 + b\cdot 15 + c = 10 \end{cases}$