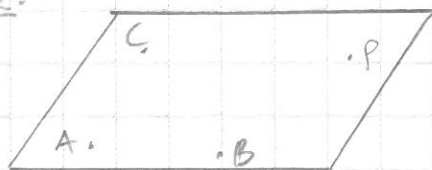


Esim. Pisteen $A = (2, -1, 1)$, $B = (2, 0, 3)$ ja $C = (4, 2, 0)$ määräävät tason. Onko piste $D = (3, -2, 2)$, $E = (2, 2, 7)$ tasolla? Missä pisteessä taso leikkaa z-akselia?

Ratk.



$$A = (2, -1, 1), B = (2, 0, 3), C = (4, 2, 0)$$

P on tasolla $\Leftrightarrow$

$$\begin{aligned}\overrightarrow{AP} &= s\overrightarrow{AB} + t\overrightarrow{AC} = s(\vec{j} + 2\vec{k}) + t(2\vec{i} + 3\vec{j} - \vec{k}) \\ &= 2t\vec{i} + (s + 3t)\vec{j} + (2s - t)\vec{k}\end{aligned}$$

$$\Rightarrow P = (2 + 2t, -1 + s + 3t, 1 + 2s - t)$$

$D = (3, -2, 2)$ on tasolla

$$\Leftrightarrow \begin{cases} 2 + 2t = 3 & (1) \\ -1 + s + 3t = -2 & (2) \\ 1 + 2s - t = 2 & (3) \end{cases}$$

$$(1) \text{ ja } (2) \Rightarrow \begin{cases} 2 + 2t = 3 \Rightarrow t = \frac{1}{2} \\ -1 + s + 3t = -2 \end{cases}$$

$$\text{Tas. (3): } 1 + 2 \cdot (-\frac{5}{2}) - \frac{1}{2} = 2 \Rightarrow 1 - 5 - \frac{1}{2} = 2 \quad \downarrow$$

$\Rightarrow D$ ei ole tasolla

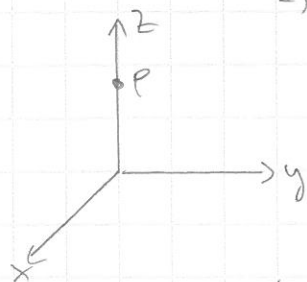
$E = (2, 2, 7)$ on tasolla

$$\Leftrightarrow \begin{cases} 2 + 2t = 2 & (1) \\ -1 + s + 3t = 2 & (2) \\ 1 + 2s - t = 7 & (3) \end{cases}$$

$$(1) \text{ ja } (2) \Rightarrow t = 0, s = 3$$

$$\text{Tas. (3): } 1 + 2 \cdot 3 - 0 = 7 \quad \checkmark$$

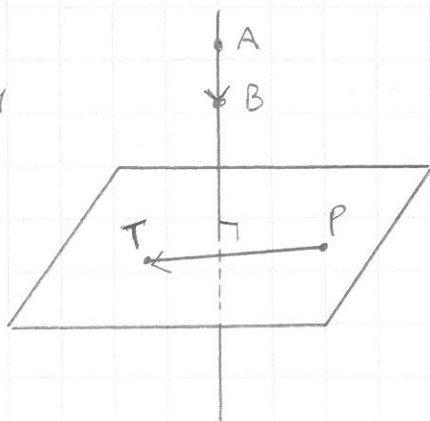
$\Rightarrow E$ on tasolla



$$P \text{ on z-akselilla} \Leftrightarrow \begin{cases} x = 2 + 2t = 0 \Rightarrow t = -1 \\ y = -1 + s + 3t = 0 \Rightarrow s = 4 \end{cases}$$

$$\Rightarrow P = (2 + 2 \cdot (-1), -1 + 4 + 3 \cdot (-1), 1 + 2 \cdot 4 - (-1)) = (0, 0, 10)$$

8.4



$$A = (5, 0, -4), B = (-2, 5, -1)$$

$$P = (1, -2, 5)$$

Piste $T = (x, y, z)$ on tasolla

$$\Leftrightarrow \overrightarrow{PT} \perp \overrightarrow{AB}$$

$$\overrightarrow{AB} = -7\vec{i} + 5\vec{j} + 3\vec{k}$$

$$\Leftrightarrow \overrightarrow{PT} \cdot \overrightarrow{AB} = 0$$

$$\overrightarrow{PT} = (x-1)\vec{i} + (y+2)\vec{j} + (z-5)\vec{k}$$

$$\Leftrightarrow -7 \cdot (x-1) + 5(y+2) + 3(z-5) = 0$$

$$\Leftrightarrow -7x + 5y + 3z + 2 = 0 \quad \text{tason normaalivuohtainen yhtälö}$$