230. tangentin kk $= f'(x) = 3x^2 - x$

$$f(x) = \int (3x^2 - 2x)\,dx = 3 \cdot \frac{1}{3} x^3 - 2 \cdot \frac{1}{2} x^2 + C$$
$$= x^3 - x^2 + C$$

$f(2) = 3$

$8 - 4 + C = 3 \Rightarrow C = -1$

V: $f(x) = x^3 - x^2 - 1$

$$\int k\,dx = kx + C$$

$$\int a \cdot f(x)\,dx = a \int f(x)\,dx$$

$$\int \left(f(x) + g(x)\right) dx = \int f(x)\,dx + \int g(x)\,dx$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$$

Yhdistetty funktio

Esim.

$f(x)$	$s(x)$	$u(x)$
$\sqrt{x+1}$	$x+1$	$\sqrt{x}$
$\sin 2x$	$2x$	$\sin x$
e^{x^2+1}	x^2+1	e^x
$(-x+1)^3$	$-x+1$	x^3
$-\dfrac{2}{x^2-x+1}$	x^2-x+1	$-\dfrac{2}{x}$

$$\boxed{D\,u(s(x)) = u'(\underset{\uparrow}{s(x)})\cdot s'(x)}$$

pitää kertoa sisäf:n derivaatalla

s(x) ei muutu

Yhdistetyn f:n integrointi

Esim.

a) $\int 1 \cdot (x+1)^2 \, dx = \frac{1}{3}(x+1)^3 + C$

$s(x) = x+1$

$s'(x) = 1$

$u(x) = x^2 \rightarrow U(x) = \frac{1}{3}x^3 + C$

$$\boxed{\int s'(x)\, u(s(x))\, dx = U(s(x)) + C}$$

b) $\frac{1}{2} \cdot \int 2 \cdot (2x+1)^2 \, dx = \frac{1}{2} \cdot \frac{1}{3}(2x+1)^3 + C$

$s(x) = 2x+1$

$s'(x) = 2$

$u(x) = x^2 \rightarrow U(x) = \frac{1}{3}x^3 + C$

c) X:ää ei saa lisätä sisäf. derivaattaan !

$$\frac{1}{2x}\int 2x\sin(x^2)\,dx$$ ei voida integroida yhd. funktion kaavalla

ei tarvitse kertoa vakiolla !

240. a) $\int 3x^2(x^3-1)^4\,dx = \frac{1}{5}(x^3-1)^5+C$

$s(x)=x^3-1$

$s'(x)=3x^2$

$u(x)=x^4 \rightarrow U(x)=\frac{1}{5}x^5+C$

$s(x)=x-1$

$s'(x)=1$

$u(x)=x^{\frac{1}{3}} \rightarrow U(x)=\frac{3}{4}x^{\frac{4}{3}}+C$

b) $\int 1\cdot\sqrt[3]{x-1}\,dx = \int 1\cdot(x-1)^{\frac{1}{3}}dx = \frac{3}{4}(x-1)^{\frac{4}{3}}+C$

ei tarvitse kertoa vakiolla kun s' on valmiiksi ok.

c) $\displaystyle\int \frac{3x}{(x^2-1)^2}\,dx = \int 3x\,(x^2-1)^{-2}\,dx$

$s(x) = x^2-1$

$s'(x) = 2x$

$u(x) = x^{-2} \rightarrow U(x) = -x^{-1}+C$

$= -\frac{1}{x}+C$

$$= 3\cdot\frac{1}{2}\cdot\int \underbrace{2\cdot x}_{s'}\;\cdot\;\underbrace{(x^2-1)^{-2}}_{u(s(x))}\,dx$$

$$= \frac{3}{2}\cdot\left(-\frac{1}{1}\right)(x^2-1)^{-1}+C$$

$$= -\frac{3}{2(x^2-1)}+C = \frac{3}{2-2x^2}+C$$

236. a) $\int 3(4x+5)^6 dx$

$s(x) = 4x+5$

$s'(x) = 4$

$u(x) = x^6 \longrightarrow U(x) = \frac{1}{7}x^7 + c$

$= 3 \cdot \frac{1}{4} \cdot \int 4 \cdot (4x+5)^6 dx$

$= \frac{3}{4} \cdot \frac{1}{7}(4x+5)^7 + C = \frac{3}{28}(4x+5)^7 + C$

c) $\int \frac{(3-5x)^4}{5} dx = \frac{1}{5} \cdot \frac{1}{-5} \int -5(3-5x)^4 dx$

$= -\frac{1}{25} \cdot \frac{1}{5}(3-5x)^5 + C$

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