Integraali kertaus

(1.) Polynomifunktioina kummatkin funktiot ovat määriteltyjä ja jatkuvia koko reaalilukujen joukossa.
Koska funktion $f(x) = x^2 + 2$ derivaatta $f'(x) = 2x = g(x)$ aina, kun $x \in \mathbb{R}$, niin funktio $f(x)$ on funktion $g(x)$ integraalifunktio.

(2.) a) $\int (2x^2 - \frac{1}{3})\,dx = 2 \cdot \frac{1}{3}x^3 - \frac{1}{3}x + C = \frac{2}{3}x^3 - \frac{1}{3}x + C$

b) $\int (2x - \frac{1}{3})^2\,dx = \int (4x^2 - \frac{4}{3}x + \frac{1}{9})\,dx = \frac{4}{3}x^3 - \frac{4}{3} \cdot \frac{1}{2}x^2 + \frac{1}{9}x + C = \frac{4}{3}x^3 - \frac{2}{3}x^2 + \frac{1}{9}x + C$

TAI $u(x) = x^2$ $s(x) = 2x$ $s'(x) = 2$

$\int (2x - \frac{1}{3})^2\,dx = \frac{1}{2}\int 2(2x - \frac{1}{3})^2\,dx = \frac{1}{2} \cdot \frac{1}{3}(2x - \frac{1}{3})^3 + C = \frac{1}{6}(2x - \frac{1}{3})^3 + C$

(3.) a) $\int \sqrt[5]{x^3}\,dx = \int x^{\frac{3}{5}}\,dx = \frac{1}{\frac{3}{5}+1}x^{\frac{3}{5}+1} + C = \frac{5}{8}x^{1\frac{3}{5}} + C = \frac{5}{8}x\sqrt[5]{x^3} + C$

b) $\int \frac{4}{x}\,dx = 4\int \frac{1}{x}\,dx = 4\ln|x| + C$

c) $\int (1-x)^2\,dx = \int (1 - 2x + x^2)\,dx = x - 2 \cdot \frac{1}{2}x^2 + \frac{1}{3}x^3 + C = \frac{1}{3}x^3 - x^2 + x + C$

d) $\int (\sin x - \cos x)\,dx = -\cos x - \sin x + C$

(4.) a) $\int (8x+1)^5\,dx$ $\{ u(x) = x^5$ $s(x) = 8x+1$ $s'(x) = 8$

$= \frac{1}{8}\int 8(8x+1)^5\,dx = \frac{1}{8} \cdot \frac{1}{6}(8x+1)^6 + C = \frac{1}{48}(8x+1)^6 + C$

b) $\int e^{-5x+2}\,dx$ $\{ u(x) = e^x$ $s(x) = -5x+2$ $s'(x) = -5$

$= -\frac{1}{5}\int (-5)e^{-5x+2}\,dx = -\frac{1}{5}e^{-5x+2} + C$

c) $\int \cos(\frac{x}{3} - \pi)\,dx$ $\{ u(x) = \cos x$ $s(x) = \frac{x}{3} - \pi$ $s'(x) = \frac{1}{3}$

$= 3\int \frac{1}{3}\cos(\frac{x}{3} - \pi)\,dx = 3\sin(\frac{x}{3} - \pi) + C$

d) $\int (9x-1)^{-1}\,dx = \int \frac{1}{9x-1}\,dx$ $\{ u(x) = \frac{1}{x}$ $s(x) = 9x-1$ $s'(x) = 9$

$= \frac{1}{9}\int \frac{9}{9x-1}\,dx = \frac{1}{9}\ln|9x-1| + C$