

12. Logaritmin laskusäännöt

$$1. \log_a x^n = n \log_a x$$

POTENSSIN LOGARITMI

$$2. \log_a (xy) = \log_a x + \log_a y$$

TULON — " —

$$3. \log_a \frac{x}{y} = \log_a x - \log_a y$$

OSAMITTOMUUS — " —

$$4. \log_a x = \frac{\log_b x}{\log_b a}$$

KANTALUVUN VAIHTO

Tod. 1. $\log_a x^n = \log_a (a^r)^n = \log_a a^{rn} = rn = n \log_a x$
 $x = a^r \Leftrightarrow r = \log_a x$

2. $\log_a (xy) = \log_a (a^t a^p) = \log_a a^{t+p} = t+p = \log_a x + \log_a y$
 $\begin{cases} x = a^t \Leftrightarrow t = \log_a x \\ y = a^p \Leftrightarrow p = \log_a y \end{cases}$

4. $\begin{cases} a^r = x \Leftrightarrow r = \log_a x \\ a^r = x \mid \log_b \Leftrightarrow \log_b a^r = \log_b x \Leftrightarrow r \log_b a = \log_b x \end{cases}$
 $\Rightarrow r = \frac{\log_b x}{\log_b a}$
 $\Rightarrow \log_a x = \frac{\log_b x}{\log_b a}$

Merk. $\log_{10} a = \lg a$

12.2 a) $\log_3 2 - \log_3 6 = \log_3 \frac{2}{6} = \log_3 \frac{1}{3} = -1 \quad (3^{-1} = \frac{1}{3})$

b) $2 \lg 2 - \lg 40 = \lg 2^2 - \lg 40 = \lg \frac{2^2}{40} = \lg \frac{4}{40} = \lg \frac{1}{10} = -1 \quad (10^{-1} = \frac{1}{10})$

12.4, 8, 9

12.4 a) $3 \log_5 x + 2 \log_5 3 - \log_5 4 = \log_5 x^3 + \log_5 3^2 - \log_5 4$
 $= \log_5 (x^3 \cdot 3^2) - \log_5 4 = \log_5 \frac{x^3 \cdot 3^2}{4} = \log_5 \frac{9x^3}{4}$

b) $1 + 2 \log_4 x = \log_4 4 + \log_4 x^2 = \log_4 (4x^2)$