7 Modelling relationships between two data sets: statistics for bivariate data

Skills check

1 a 1296 **b** 64 **c** 343

2 a 5 **b** 4 **c** 3 **d** 3

3 a $y = 4x - 2$ **b** $y = 1 - 2x$

Exercise 7A

1 a There is a strong, positive, linear correlation

b There is a weak, negative, linear correlation

c There is a strong, negative, linear correlation

d There is a weak, positive, linear correlation

e There is no correlation

2 i a Positive **b** Linear
c Strong

ii a Positive **b** Linear
c Moderate

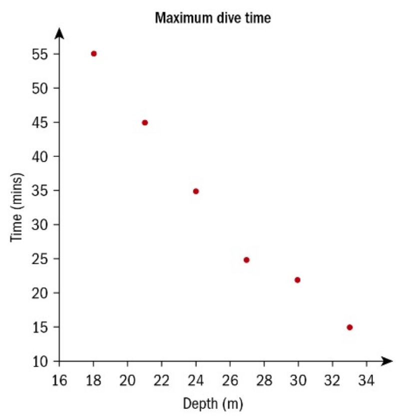
iii a Positive **b** Linear
c Weak

iv a No correlation **b** Non linear
c Zero

v a Negative **b** Linear
c Strong

vi a Negative **b** Non linear
c Strong

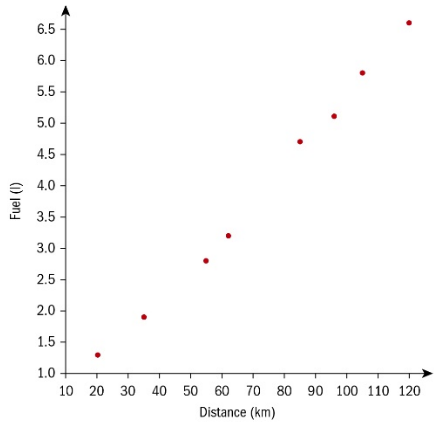
3 a



b There is a strong, negative, linear correlation

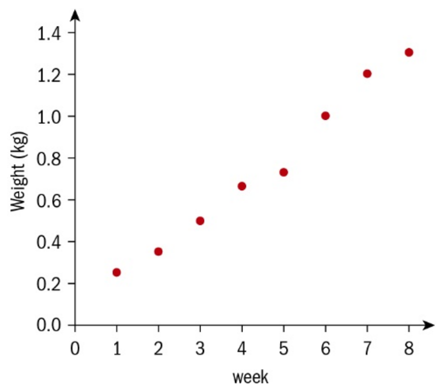
c As the maximum depth increases the time at that depth decreases

4 a



b There is a strong, positive, linear correlation

5 a



b There is a strong, positive, linear correlation

c As the kitten gets older, it gets heavier

Exercise 7B

1 a

x	y	x^2	y^2	xy
20	250	400	62500	5000
24	300	576	90000	7200
30	350	900	122500	10500
40	360	1600	129600	14400
50	480	2500	230400	24000
75	580	5625	336400	43500
80	750	6400	562500	60000
90	840	8100	705600	75600
100	900	10000	810000	90000
120	1000	14400	1000000	120000
$\Sigma x = 629$	$\Sigma y = 5810$	$\Sigma x^2 = 50501$	$\Sigma y^2 = 4049500$	$\Sigma xy = 450200$

$$S_{xx} = \Sigma x^2 - \frac{(\Sigma x)^2}{n}$$

$$S_{xx} = 50501 - \frac{629^2}{10} = \frac{109369}{10}$$

$$S_{yy} = \Sigma y^2 - \frac{(\Sigma y)^2}{n}$$

$$S_{yy} = 4049500 - \frac{5810^2}{10} = 673890$$

$$S_{xy} = \Sigma xy - \frac{(\Sigma x)(\Sigma y)}{n}$$

$$S_{xy} = 450200 - \frac{629 \times 5810}{10} = 84751$$

$$r = \frac{S_{xy}}{\sqrt{(S_{xx}S_{yy})}}$$

$$r = \frac{84751}{\sqrt{\frac{109369}{10} \times 673890}} = 0.987$$

b There is a strong positive correlation

c As the floor area increases, house price increases

2 a

x	y	x^2	y^2	xy
1	40000	1	1600000000	40000
2	36500	4	1332250000	73000
3	31000	9	961000000	93000
4	26658	16	710648954	106632
5	24250	25	588062500	121250
6	19540	36	381811600	117240
7	19100	49	364810000	133700
8	18750	64	351562500	150000
9	15430	81	238084900	138870
10	12600	100	158760000	126000
$\Sigma x =$ 55	$\Sigma y =$ 243828	$\Sigma x^2 =$ 385	$\Sigma y^2 =$ 6686990464	$\Sigma xy =$ 1099692

$$S_{xx} = \Sigma x^2 - \frac{(\Sigma x)^2}{n}$$

$$S_{xx} = 385 - \frac{55^2}{10} = \frac{165}{2}$$

$$S_{yy} = \Sigma y^2 - \frac{(\Sigma y)^2}{n}$$

$$S_{yy} = 6686990464 - \frac{243828^2}{10}$$

$$= \frac{3708905528}{5}$$

$$S_{xy} = \Sigma xy - \frac{(\Sigma x)(\Sigma y)}{n}$$

$$S_{xy} = 1099692 - \frac{55 \times 243828}{10}$$

$$= -241362$$

$$r = \frac{S_{xy}}{\sqrt{(S_{xx}S_{yy})}}$$

$$r = \frac{-241362}{\sqrt{\frac{165}{2} \times \frac{3708905528}{5}}} = -0.976$$

b There is a strong negative correlation

c The price of the motorbike can never fall below 0.

3 a

x	y	x^2	y^2	xy
148	34	21904	1156	5032
153	38	23409	1444	5814
165	42	27225	1764	6930
142	36	20164	1296	5112
155	42	24025	1764	6510
141	32	19881	1024	4512
171	40	29241	1600	6840
154	34	23716	1156	5236
170	40	28900	1600	6800
168	38	28224	1444	6384
Σx = 1567	Σy = 376	Σx^2 = 246689	Σy^2 = 14248	Σxy = 59170

$$S_{xx} = \Sigma x^2 - \frac{(\Sigma x)^2}{n}$$

$$S_{xx} = 246689 - \frac{1567^2}{10} = \frac{11401}{10}$$

$$S_{yy} = \Sigma y^2 - \frac{(\Sigma y)^2}{n}$$

$$S_{yy} = 14248 - \frac{376^2}{10} = \frac{552}{5}$$

$$S_{xy} = \Sigma xy - \frac{(\Sigma x)(\Sigma y)}{n}$$

$$S_{xy} = 59170 - \frac{1567 \times 376}{10} = \frac{1254}{5}$$

$$r = \frac{S_{xy}}{\sqrt{(S_{xx}S_{yy})}}$$

$$r = \frac{\frac{1254}{5}}{\sqrt{\frac{11401}{10} \times \frac{552}{5}}} = 0.707$$

b There is a moderate positive correlation

4 a

x	y	x^2	y^2	xy
6	78	36	6084	468
4	80	16	6400	320
7	86	49	7396	602
5	88	25	7744	440
1	66	1	4356	66
2	70	4	4900	140
4	78	16	6084	312
6	95	36	9025	570
8	97	64	9409	776
4	76	16	5776	304
$\Sigma x = 47$	$\Sigma y = 814$	$\Sigma x^2 = 263$	$\Sigma y^2 = 67174$	$\Sigma xy = 3998$

$$S_{xx} = \Sigma x^2 - \frac{(\Sigma x)^2}{n}$$

$$S_{xx} = 263 - \frac{47^2}{10} = \frac{421}{10}$$

$$S_{yy} = \Sigma y^2 - \frac{(\Sigma y)^2}{n}$$

$$S_{yy} = 67174 - \frac{814^2}{10} = \frac{4572}{5}$$

$$S_{xy} = \Sigma xy - \frac{(\Sigma x)(\Sigma y)}{n}$$

$$S_{xy} = 3998 - \frac{47 \times 814}{10} = \frac{861}{5}$$

$$r = \frac{S_{xy}}{\sqrt{(S_{xx}S_{yy})}}$$

$$r = \frac{\frac{861}{5}}{\sqrt{\frac{421}{10} \times \frac{4572}{5}}} = 0.878$$

b There is a strong positive correlation

c Yes

5 a

x	y	x^2	y^2	xy
3.9	10	15.21	100	39
2.7	14	7.29	196	37.8
3.8	5	14.44	25	19
2.4	8	5.76	64	19.2
1.7	24	2.89	576	40.8
2.6	17	6.76	289	44.2
4.0	21	16	441	84
3.7	7	13.69	49	25.9
$\Sigma x = 24.8$	$\Sigma y = 106$	$\Sigma x^2 = 82.04$	$\Sigma y^2 = 1740$	$\Sigma xy = 309.9$

$$S_{xx} = \Sigma x^2 - \frac{(\Sigma x)^2}{n}$$

$$S_{xx} = 82.04 - \frac{24.8^2}{8} = 5.16$$

$$S_{yy} = \Sigma y^2 - \frac{(\Sigma y)^2}{n}$$

$$S_{yy} = 1740 - \frac{106^2}{8} = 335.5$$

$$S_{xy} = \Sigma xy - \frac{(\Sigma x)(\Sigma y)}{n}$$

$$S_{xy} = 309.9 - \frac{24.8 \times 106}{8} = -18.7$$

$$r = \frac{S_{xy}}{\sqrt{(S_{xx}S_{yy})}}$$

$$r = \frac{-18.7}{\sqrt{5.16 \times 335.5}} = -0.449$$

b There is a weak negative correlation

c Yes, because the correlation is only weak

6 a

x	y	x^2	y^2	xy
52	60	2704	3600	3120
60	68	3600	4624	4080
62	66	3844	4356	4092
65	69	4225	4761	4485
68	75	4624	5625	5100
76	82	5776	6724	6232
77	83	5929	6889	6391
78	84	6084	7056	6552
80	88	6400	7744	7040
84	90	7056	8100	7560
85	93	7225	8649	7905
95	92	9025	8464	8740
$\Sigma x = 882$	$\Sigma y = 950$	$\Sigma x^2 = 66492$	$\Sigma y^2 = 76592$	$\Sigma xy = 71297$

$$S_{xx} = \Sigma x^2 - \frac{(\Sigma x)^2}{n}$$

$$S_{xx} = 66492 - \frac{882^2}{12} = 1665$$

$$S_{yy} = \Sigma y^2 - \frac{(\Sigma y)^2}{n}$$

$$S_{yy} = 76592 - \frac{950^2}{12} = 1383.67$$

$$S_{xy} = \Sigma xy - \frac{(\Sigma x)(\Sigma y)}{n}$$

$$S_{xy} = 71297 - \frac{882 \times 950}{12} = 1472$$

$$r = \frac{S_{xy}}{\sqrt{(S_{xx}S_{yy})}}$$

$$r = \frac{1472}{\sqrt{1665 \times 1383.67}} = 0.970$$

b There is a strong positive correlation

c More practice questions will likely increase the overall grade

Exercise 7C

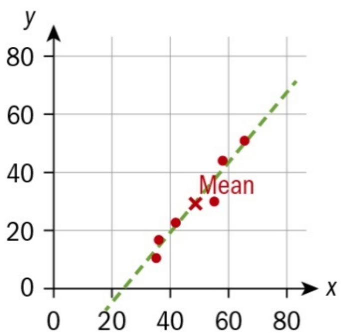
1 a mean $= \dfrac{36+55+42+35+58+65}{6}$

$= 48.5$

b mean $= \dfrac{17+30+23+11+44+51}{6}$

$= 29.333$

c



2 a mean age

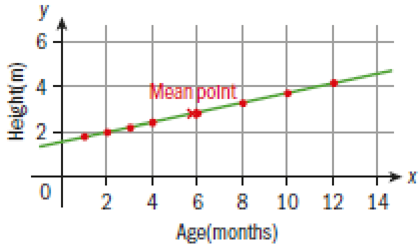
$= \dfrac{1+2+3+4+6+8+10+12}{8} = 5.75$

mean height

$= \dfrac{1.78+1.98+2.17+2.40+2.82+3.26+3.71+4.14}{8}$

$= 2.7825$

b



c $y - y_1 = \dfrac{y_2 - y_1}{x_2 - x_1} \times (x - x_1)$

$y - 2.7825 = \dfrac{2.7825 - 2}{5.75 - 2} \times (x - 5.75)$

$y = 0.209x + 1.583$

d $y = 0.209 \times 9 + 1.583 = 3.46$ m

e $y = 0.209 \times 120 + 1.583 = 26.6$ m

f Not reliable as it is known that giraffes only grow to 6 m

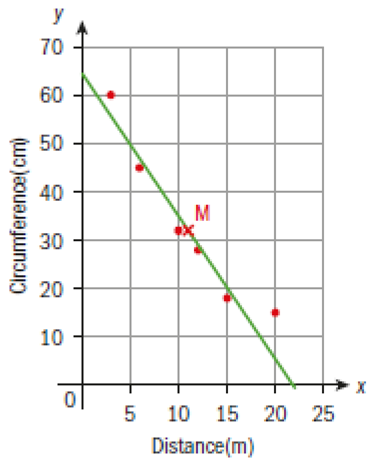
3 a mean $= \dfrac{3+6+10+12+15+20}{6}$

$= 11$ km

b mean $= \dfrac{60+45+32+28+18+15}{6}$

$= 33{,}000$ Rupees

c



d $y - y_1 = \dfrac{y_2 - y_1}{x_2 - x_1} \times (x - x_1)$

$y - 33 = \dfrac{33 - 50}{11 - 5} \times (x - 11)$

$y = -2.833x + 64.1667$

e $y = -2.833 \times 8 + 64.1667 = 41.5$

$\Rightarrow$ 41,500 Rupees

f $50 = -2.833x + 64.1667$

$\Rightarrow x = \dfrac{50 - 64.1667}{-2.833} = 5$ km

g $y = -2.833 \times 30 + 64.1667$

$= -20.8233$

$\Rightarrow -20,823$ Rupees

Not suitable to extrapolate, negative rent is not correct

Exercise 7D

1 A student who plays no sport will spend 35 hours on homework and each day spent playing sport reduces the hours of homework by 30 minutes

2 A person who has no friends who are criminals has 1 conviction and adding one extra criminal friends leads to 6 extra convictions

3 A brand new speaker is worth \$300 and as it gets older, its value decreases by \$40 per year

4 a

x	y	x^2	y^2	xy
6	157	36	24649	942
7	155	49	24025	1085
8	147	64	21609	1176
8.5	142	72.25	20164	1207
9	138	81	19044	1242
9.5	132	90.25	17424	1254
10	134	100	17956	1340
11	127	121	16129	1397
11.5	120	132.25	14400	1380
12	115	144	13225	1380
$\Sigma x = 92.5$	$\Sigma y = 1367$	$\Sigma x^2 = 889.75$	$\Sigma y^2 = 188625$	$\Sigma xy = 12403$

$$S_{xx} = \Sigma x^2 - \frac{(\Sigma x)^2}{n}$$

$$S_{xx} = 889.75 - \frac{92.5^2}{10} = 34.125$$

$$S_{yy} = \Sigma y^2 - \frac{(\Sigma y)^2}{n}$$

$$S_{yy} = 188625 - \frac{1367^2}{10} = 1756.1$$

$$S_{xy} = \Sigma xy - \frac{(\Sigma x)(\Sigma y)}{n}$$

$$S_{xy} = 12403 - \frac{92.5 \times 1367}{10} = -241.75$$

$$r = \frac{S_{xy}}{\sqrt{(S_{xx}S_{yy})}}$$

$$r = \frac{-241.75}{\sqrt{34.125 \times 1756.1}} = -0.988$$

b $y = a + bx$, where

$b = \dfrac{S_{xy}}{S_{xx}} = \dfrac{-241.75}{34.125} = -7.084$ and

$a = \bar{y} - b\bar{x} = \dfrac{1367}{10} + 7.084 \times \dfrac{92.5}{10} = 202.227$

so $y = 202 - 7.084x$

c $y = 202.227 - 7.084 \times 7.5 = 149 \text{ kmh}^{-1}$

d As the time taken to accelerate from 0 to 90 increases by 1 second, the top speed decreases by 7.08

5 a

x	y	x^2	y^2	xy
80	74	6400	5476	5920
73	62	5329	3844	4526
95	93	9025	8649	8835
84	75	7056	5625	6300
67	73	4489	5329	4891
88	81	7744	6561	7128
69	58	4761	3364	4002
92	90	8464	8100	8280
90	84	8100	7056	7560
$\Sigma x = 738$	$\Sigma y = 690$	$\Sigma x^2 = 61368$	$\Sigma y^2 = 54004$	$\Sigma xy = 57442$

$$S_{xx} = \Sigma x^2 - \frac{(\Sigma x)^2}{n}$$

$$S_{xx} = 61368 - \frac{738^2}{9} = 852$$

$$S_{yy} = \Sigma y^2 - \frac{(\Sigma y)^2}{n}$$

$$S_{yy} = 54004 - \frac{690^2}{9} = 1104$$

$$S_{xy} = \Sigma xy - \frac{(\Sigma x)(\Sigma y)}{n}$$

$$S_{xy} = 57442 - \frac{738 \times 690}{9} = 862$$

$$r = \frac{S_{xy}}{\sqrt{(S_{xx}S_{yy})}}$$

$$r = \frac{862}{\sqrt{852 \times 1104}} = 0.889$$

b $y = a + bx$, where

$b = \dfrac{S_{xy}}{S_{xx}} = \dfrac{862}{852} = 1.012$ and

$a = \bar{y} - b\bar{x} = \dfrac{690}{9} - 1.012 \times \dfrac{738}{9} = -6.30$

so $y = 1.01x - 6.30$

c $y = 1.012 \times 75 - 6.30 = 69.6$

6 a

x	y	x^2	y^2	xy
35	13	1225	169	455
38	18	1444	324	684
42	27	1764	729	1134
45	28	2025	784	1260
47	36	2209	1296	1692
48	34	2304	1156	1632
50	40	2500	1600	2000
$\Sigma x = 305$	$\Sigma y = 196$	$\Sigma x^2 = 13471$	$\Sigma y^2 = 6058$	$\Sigma xy = 8857$

$$S_{xx} = \Sigma x^2 - \frac{(\Sigma x)^2}{n}$$

$$S_{xx} = 13471 - \frac{305^2}{7} = 181.714$$

$$S_{yy} = \Sigma y^2 - \frac{(\Sigma y)^2}{n}$$

$$S_{yy} = 6058 - \frac{196^2}{7} = 570$$

$$S_{xy} = \Sigma xy - \frac{(\Sigma x)(\Sigma y)}{n}$$

$$S_{xy} = 8857 - \frac{305 \times 196}{7} = 317$$

$$r = \frac{S_{xy}}{\sqrt{(S_{xx}S_{yy})}}$$

$$r = \frac{317}{\sqrt{181.714 \times 570}} = 0.985$$

There is a strong, positive correlation

b $y = a + bx$, where

$$b = \frac{S_{xy}}{S_{xx}} = \frac{317}{181.714} = 1.744 \text{ and}$$

$$a = \bar{y} - b\bar{x} = \frac{196}{7} - 1.744 \times \frac{305}{7} = -48.0321$$

so $y = 1.74x - 48.0$

c $y = 1.74 \times 40 - 48.0 = 21.6$ cm

d For every cm that the cat grows in length, it grows 1.74 cm in height

7 a $a = -8.46$ and $b = 33.0$

b $y = 32.95 - 8.46 \times 3.5 = 3.34$

$\Rightarrow$ 3 mudbugs

c

x	y	x^2	y^2	xy
0.5	30	0.25	900	15
1	28	1	784	28
1.5	14	2.25	196	21
2	18	4	324	36
2.5	10	6.25	100	25
3	7	9	49	21
4	1	16	1	4
$\Sigma x = 14.5$	$\Sigma y = 108$	$\Sigma x^2 = 38.75$	$\Sigma y^2 = 2354$	$\Sigma xy = 150$

$$S_{xx} = \Sigma x^2 - \frac{(\Sigma x)^2}{n}$$

$$S_{xx} = 38.75 - \frac{14.5^2}{7} = 8.714$$

$$S_{yy} = \Sigma y^2 - \frac{(\Sigma y)^2}{n}$$

$$S_{yy} = 2354 - \frac{108^2}{7} = 687.714$$

$$S_{xy} = \Sigma xy - \frac{(\Sigma x)(\Sigma y)}{n}$$

$$S_{xy} = 150 - \frac{14.5 \times 108}{7} = -73.714$$

$$r = \frac{S_{xy}}{\sqrt{(S_{xx}S_{yy})}}$$

$$r = \frac{-73.714}{\sqrt{8.714 \times 687.714}} = -0.952$$

d Strong, negative

8 a

x	y	x^2	y^2	xy
28	66	784	4356	1848
33	70	1089	4900	2310
35	85	1225	7225	2975
42	94	1764	8836	3948
40	96	1600	9216	3840
38	80	1444	6400	3040
$\Sigma x = 216$	$\Sigma y = 491$	$\Sigma x^2 = 7906$	$\Sigma y^2 = 40933$	$\Sigma xy = 17961$

$$S_{xx} = \Sigma x^2 - \frac{(\Sigma x)^2}{n}$$

$$S_{xx} = 7906 - \frac{216^2}{6} = 130$$

$$S_{yy} = \Sigma y^2 - \frac{(\Sigma y)^2}{n}$$

$$S_{yy} = 40933 - \frac{491^2}{6} = 752.833$$

$$S_{xy} = \Sigma xy - \frac{(\Sigma x)(\Sigma y)}{n}$$

$$S_{xy} = 17961 - \frac{216 \times 491}{6} = 285$$

$$r = \frac{S_{xy}}{\sqrt{(S_{xx}S_{yy})}}$$

$$r = \frac{285}{\sqrt{130 \times 752.833}} = 0.911$$

b $b = \frac{S_{xy}}{S_{xx}} = \frac{285}{130} = 2.19$ and

$$a = \bar{y} - b\bar{x} = \frac{491}{6} - 2.192 \times \frac{216}{6} = 2.92$$

c If a student scores 1 mark better in the IB diploma then they will do 2.19% better in their first year at university

d $y = 2.19x + 2.92$

$= 2.19 \times 30 + 2.92 = 68.7\%$

9 a

x	y	x^2	y^2	xy
25	200	625	40000	5000
40	260	1600	67600	10400
65	350	4225	122500	22750
53	360	2809	129600	19080
46	260	2116	67600	11960
30	250	900	62500	7500
50	310	2500	96100	15500
74	600	5476	360000	44400
70	450	4900	202500	31500
$\Sigma x = 453$	$\Sigma y = 3040$	$\Sigma x^2 = 25151$	$\Sigma y^2 = 1148400$	$\Sigma xy = 168090$

$$S_{xx} = \Sigma x^2 - \frac{(\Sigma x)^2}{n}$$

$$S_{xx} = 25151 - \frac{453^2}{9} = 2350$$

$$S_{yy} = \Sigma y^2 - \frac{(\Sigma y)^2}{n}$$

$$S_{yy} = 1148100 - \frac{3040^2}{9} = 121256$$

$$S_{xy} = \Sigma xy - \frac{(\Sigma x)(\Sigma y)}{n}$$

$$S_{xy} = 168090 - \frac{453 \times 3040}{9} = 15076.7$$

$y = a + bx$, where

$$b = \frac{S_{xy}}{S_{xx}} = \frac{15076.7}{2350} = 6.416 \text{ and}$$

$$a = \bar{y} - b\bar{x} = \frac{3040}{9} - 6.416 \times \frac{453}{9} = 14.858$$

so $y = 6.416x + 14.858$

b **i** Each additional pizza costs \$6.42

ii When no pizzas are made, there is a cost of \$14.86

c $y = 6.416 \times 60 + 14.858 = \$399.82$

d **i** Not reliable as 5000 is not close to the domain used

ii $100 = 6.416x + 14.858$

$$x = \frac{100 - 14.858}{6.416}$$

$x = 13.27$

13 pizzas

10 a

x	y	x^2	y^2	xy
1	115	1	13225	115
2	110	4	12100	220
3	92	9	8464	276
4	89	16	7921	356
5	80	25	6400	400
8	63	64	3969	504
9	59	81	3481	531
10	54	100	2916	540
$\Sigma x = 42$	$\Sigma y = 662$	$\Sigma x^2 = 300$	$\Sigma y^2 = 58476$	$\Sigma xy = 2942$

$$S_{xx} = \Sigma x^2 - \frac{(\Sigma x)^2}{n}$$

$$S_{xx} = 300 - \frac{42^2}{8} = 79.5$$

$$S_{yy} = \Sigma y^2 - \frac{(\Sigma y)^2}{n}$$

$$S_{yy} = 58476 - \frac{662^2}{8} = 3695.5$$

$$S_{xy} = \Sigma xy - \frac{(\Sigma x)(\Sigma y)}{n}$$

$$S_{xy} = 2942 - \frac{42 \times 662}{8} = -533.5$$

$$r = \frac{S_{xy}}{\sqrt{(S_{xx}S_{yy})}}$$

$$r = \frac{-533.5}{\sqrt{79.5 \times 3695.5}} = -0.984$$

b $b = \frac{S_{xy}}{S_{xx}} = \frac{-533.5}{79.5} = -6.71$ and

$$a = \bar{y} - b\bar{x} = \frac{662}{8} + 6.711 \times \frac{42}{8} = 117.98$$

c $y = 117.98 - 6.711 \times 6 = 77.717$

$= ¥78000$

Exercise 7E

1 a

x	y	x^2	y^2	xy
12	45	144	2025	540
15	44	225	1936	660
18	45	324	2025	810
18	42	324	1764	756
22	40	484	1600	880
25	34	625	1156	850
30	26	900	676	780
$\Sigma x = 140$	$\Sigma y = 276$	$\Sigma x^2 = 3026$	$\Sigma y^2 = 11182$	$\Sigma xy = 5276$

$$S_{xx} = \Sigma x^2 - \frac{(\Sigma x)^2}{n}$$

$$S_{xx} = 3026 - \frac{140^2}{7} = 226$$

$$S_{yy} = \Sigma y^2 - \frac{(\Sigma y)^2}{n}$$

$$S_{yy} = 11182 - \frac{276^2}{7} = 299.714$$

$$S_{xy} = \Sigma xy - \frac{(\Sigma x)(\Sigma y)}{n}$$

$$S_{xy} = 5276 - \frac{140 \times 276}{7} = -244$$

$y = a + bx$, where

$$b = \frac{S_{xy}}{S_{xx}} = \frac{-244}{226} = -1.0797 \text{ and}$$

$$a = \bar{y} - b\bar{x} = \frac{276}{7} + 1.0797 \times \frac{140}{7} = 61.0226$$

so $y = 61.0 - 1.08x$,

$y = 61.0 - 1.08 \times 20 = 39.4 \Rightarrow 39$ tickets

b $x = a + by$, where

$$b = \frac{S_{xy}}{S_{yy}} = \frac{-244}{299.714} = -0.814 \text{ and}$$

$$a = \bar{x} - b\bar{y} = \frac{140}{7} + 0.814 \times \frac{276}{7} = 52.1$$

so $x = 52.1 - 0.814y$,

$y = 52.1 - 0.814 \times 35 = 23.61 \Rightarrow \$24$

2 a

x	y	x^2	y^2	xy
2	6	4	36	12
3	10	9	100	30
5	22	25	484	110
7	33	49	1089	231
8	42	64	1764	336
10	56	100	3136	560
$\Sigma x = 35$	$\Sigma y = 169$	$\Sigma x^2 = 251$	$\Sigma y^2 = 6609$	$\Sigma xy = 1279$

$$S_{yy} = \Sigma y^2 - \frac{(\Sigma y)^2}{n}$$

$$S_{yy} = 6609 - \frac{169^2}{6} = 1848.83$$

$$S_{xy} = \Sigma xy - \frac{(\Sigma x)(\Sigma y)}{n}$$

$$S_{xy} = 1279 - \frac{35 \times 169}{6} = 293.167$$

$x = a + by$, where

$$b = \frac{S_{xy}}{S_{yy}} = \frac{293.167}{1848.83} = 0.159 \text{ and}$$

$$a = \bar{x} - b\bar{y} = \frac{35}{6} - 0.159 \times \frac{169}{6} = 1.35 \text{ so}$$

$x = 1.35 + 0.159y$

b $x = 1.35 + 0.159 \times 50 = 9.3$ mins

3

x	y	x^2	y^2	xy
90	87	8100	7569	7830
88	57	7744	3249	5061
65	52	4225	2704	3380
92	76	8464	5776	6992
50	30	2500	900	1500
67	67	4489	4489	4489
100	96	10000	9216	9600
100	74	10000	5476	7400
73	65	5329	4225	4745
90	87	8100	7569	7830
83	78	6889	6084	6474
94	89	8836	7921	8366
83	78	6889	6084	6474
$\Sigma x = 1075$	$\Sigma y = 936$	$\Sigma x^2 = 91565$	$\Sigma y^2 = 71262$	$\Sigma xy = 80096$

$$S_{yy} = \Sigma y^2 - \frac{(\Sigma y)^2}{n}$$

$$S_{yy} = 71262 - \frac{936^2}{13} = 3870$$

$$S_{xy} = \Sigma xy - \frac{(\Sigma x)(\Sigma y)}{n}$$

$$S_{xy} = 80096 - \frac{1075 \times 936}{13} = 2696$$

$x = a + by$, where

$$b = \frac{S_{xy}}{S_{yy}} = \frac{2696}{3870} = 0.697 \text{ and}$$

$$a = \bar{x} - b\bar{y} = \frac{1075}{13} - 0.697 \times \frac{936}{13} = 32.508$$

so $x = 32.5 + 0.697y$,

$x = 32.508 + 0.697 \times 52 = 68.752$

$\Rightarrow$ 69 marks in mathematics

4 a

x	**y**	**x²**	**y²**	**xy**
1	180	1	32400	180
5	164	25	26896	820
9	148	81	21904	1332
12	120	144	14400	1440
14	118	196	13924	1652
19	90	361	8100	1710
21	85	441	7225	1785
24	82	576	6724	1968
30	65	900	4225	1950
34	60	1156	3600	2040
$\Sigma x = 169$	$\Sigma y = 1112$	$\Sigma x^2 = 3881$	$\Sigma y^2 = 139398$	$\Sigma xy = 14877$

$$S_{xx} = \Sigma x^2 - \frac{(\Sigma x)^2}{n}$$

$$S_{xx} = 3881 - \frac{169^2}{10} = 1024.9$$

$$S_{yy} = \Sigma y^2 - \frac{(\Sigma y)^2}{n}$$

$$S_{yy} = 139398 - \frac{1112^2}{10} = 15743.6$$

$$S_{xy} = \Sigma xy - \frac{(\Sigma x)(\Sigma y)}{n}$$

$$S_{xy} = 14877 - \frac{169 \times 1112}{10} = -3915.8$$

$y = a + bx$, where

$$b = \frac{S_{xy}}{S_{xx}} = \frac{-3915.8}{1024.9} = -3.821 \text{ and}$$

$$a = \bar{y} - b\bar{x} = \frac{1112}{10} + 3.821 \times \frac{169}{10} = 175.775$$

so $y = 175.775 - 3.821x$,

$y = 175.775 - 3.821 \times 7 = 149.028 = 149$

b $x = a + by$, where

$$b = \frac{S_{xy}}{S_{yy}} = \frac{-3915.8}{15743.6} = -0.249 \text{ and}$$

$$a = \bar{x} - b\bar{y} = \frac{169}{10} + 0.249 \times \frac{1112}{10} = 44.589$$

so $x = 44.589 - 0.249y$,

$x = 44.589 - 0.249 \times 100 = 19.7$ km

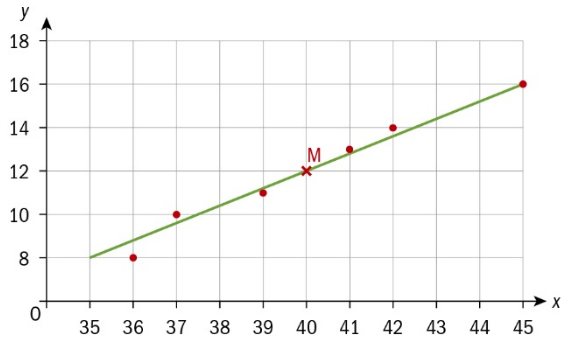
Chapter Review

1 a The PMCC lies between -1 and 1

b A -0.6, B 0.9, C 0.5, D 0, E -0.96

c Strong negative, linear

2 a,d & f



b mean temp

$$= \frac{39 + 36 + 45 + 41 + 42 + 37}{6} = \frac{240}{6}$$

$= 40$ °C

c mean cost

$$= \frac{11 + 8 + 16 + 13 + 14 + 10}{6} = \frac{72}{6} = 12$$

so 1200 Dirhams

e $$y - y_1 = \frac{y_1 - y_2}{x_1 - x_2}(x - x_1)$$

$$y - 12 = \frac{12 - 16}{40 - 45}(x - 40)$$

$$y = 0.8x - 32 + 12$$

$$y = 0.8x - 20$$

3 a

t	**e**	**t²**	**e²**	**te**
0	29	0	841	0
2	38	4	1444	76
4	27	16	729	108
6	19	36	361	114
8	12	64	144	96
$\Sigma t = 20$	$\Sigma e = 125$	$\Sigma t^2 = 120$	$\Sigma e^2 = 3519$	$\Sigma te = 394$

$$S_{tt} = \Sigma t^2 - \frac{(\Sigma t)^2}{n}$$

$$S_{tt} = 120 - \frac{20^2}{5} = 40$$

$$S_{ee} = \Sigma e^2 - \frac{(\Sigma e)^2}{n}$$

$$S_{ee} = 3519 - \frac{125^2}{5} = 394$$

$$S_{te} = \Sigma te - \frac{(\Sigma t)(\Sigma e)}{n}$$

$$S_{te} = 394 - \frac{20 \times 125}{5} = -106$$

$e = at + b$, where

$a = \frac{S_{te}}{S_{tt}} = \frac{-106}{40} = -2.65$ and

$b = \bar{e} - a\bar{t} = \frac{125}{5} + 2.65 \times \frac{20}{5} = 35.6$

b $e = 35.6 - 2.65 \times 5 = 22.35 \Rightarrow 22$ eggs

c Because $t = 40$ is too far outside the domain

4 a

x	y	x^2	y^2	xy
28	3600	784	12960000	100800
46	5200	2116	27040000	239200
38	4400	1444	19360000	167200
34	3800	1156	14440000	129200
52	6000	2704	36000000	312000
50	5900	2500	34810000	295000
$\Sigma x = 248$	$\Sigma y = 28900$	$\Sigma x^2 = 10704$	$\Sigma y^2 = 144610000$	$\Sigma xy = 1243400$

$$S_{xx} = \Sigma x^2 - \frac{(\Sigma x)^2}{n}$$

$$S_{xx} = 10704 - \frac{248^2}{6} = 453.333$$

$$S_{yy} = \Sigma y^2 - \frac{(\Sigma y)^2}{n}$$

$$S_{yy} = 144610000 - \frac{28900^2}{6} = 5408333$$

$$S_{xy} = \Sigma xy - \frac{(\Sigma x)(\Sigma y)}{n}$$

$$S_{xy} = 1243400 - \frac{248 \times 28900}{6} = 48866.7$$

$y = a + bx$, where

$b = \frac{S_{xy}}{S_{xx}} = \frac{48866.7}{453.333} = 107.794$ and

$a = \bar{y} - b\bar{x} = \frac{28900}{6} - 107.794 \times \frac{248}{6} = 361.181$

so $y = 108x + 361$

b Need to find the smallest x such that $120x > 107.794x + 361.181$,

$$120x > 107.794x + 361.181$$
$$(120 - 107.794)x > 361.181$$
$$x > \frac{361.181}{12.206}$$
$$x > 29.59$$

So the smallest number of chairs is 30

5 a $\frac{24 + 23.5 + 23 + 22 + 21 + 20.3 + 20 + 18.2 + 17 + 26}{10}$

$= \frac{215}{10} = 21.5$

b

x	y	x^2	y^2	xy
24	260	576	67600	6240
23.5	199	552.25	39601	4676.5
23	174	529	30276	4002
22	162	484	26244	3564
21	149	441	22201	3129
20.3	135	412.09	18225	2740.5
20	118	400	13924	2360
18.2	115	331.24	13225	2093
17	102	289	10404	1734
26	246	676	60516	6396
$\Sigma x = 215$	$\Sigma y = 1660$	$\Sigma x^2 = 4690.58$	$\Sigma y^2 = 302216$	$\Sigma xy = 36935$

$$S_{xx} = \Sigma x^2 - \frac{(\Sigma x)^2}{n}$$

$$S_{xx} = 4690.58 - \frac{215^2}{10} = 68.08$$

$$S_{yy} = \Sigma y^2 - \frac{(\Sigma y)^2}{n}$$

$$S_{yy} = 302216 - \frac{1660^2}{10} = 26656$$

$$S_{xy} = \Sigma xy - \frac{(\Sigma x)(\Sigma y)}{n}$$

$$S_{xy} = 36935 - \frac{215 \times 1660}{10} = 1245$$

$$r = \frac{S_{xy}}{\sqrt{(S_{xx}S_{yy})}}$$

$$r = \frac{1245}{\sqrt{68.08 \times 26656}} = 0.924$$

c There is a strong positive correlation. The hotter the day, the more bottles sold.

d $y = a + bx$, where

$b = \frac{S_{xy}}{S_{xx}} = \frac{1245}{68.08} = 18.29$ and

$a = \bar{y} - b\bar{x} = \frac{1660}{10} - 18.29 \times \frac{215}{10} = -227.235$

so $y = 18.3x - 227$

e $y = 18.29 \times 19.6 - 227.235 = 131.249$

$\Rightarrow 131$ bottles

f 36 is far outside the domain that we have

6 a

x	y	x^2	y^2	xy
3500	110000	12250000	12100000000	385000000
2000	65000	4000000	4225000000	130000000
5000	100000	25000000	10000000000	500000000
6000	135000	36000000	18225000000	810000000
5000	120000	25000000	14400000000	600000000
3000	90000	9000000	8100000000	270000000
4000	100000	16000000	10000000000	400000000
8000	140000	64000000	19600000000	1120000000
$\Sigma x =$ 36500	$\Sigma y =$ 860000	$\Sigma x^2 =$ 191250000	$\Sigma y^2 =$ 96650000000	$\Sigma xy =$ 4215000000

$$S_{xx} = \Sigma x^2 - \frac{(\Sigma x)^2}{n}$$

$$S_{xx} = 191250000 - \frac{36500^2}{8}$$

$$= 24718750$$

$$S_{yy} = \Sigma y^2 - \frac{(\Sigma y)^2}{n}$$

$$S_{yy} = 96650000000 - \frac{860000^2}{8}$$

$$= 4200000000$$

$$S_{xy} = \Sigma xy - \frac{(\Sigma x)(\Sigma y)}{n}$$

$$S_{xy} = 4215000000 - \frac{36500 \times 860000}{8}$$

$$= 291250000$$

$y = a + bx$, where

$$b = \frac{S_{xy}}{S_{xx}} = \frac{291250000}{24718750} = 11.78 \text{ and}$$

$$a = \bar{y} - b\bar{x} = \frac{860000}{8} - 11.78 \times \frac{36500}{8} = 53753.8$$

so $y = 11.8x + 53754$

b $y = 11.8 \times 7000 + 53754 = \$136354$

c i and ii would change, iii would remain the same

7 a

x	y	x^2	y^2	xy
30	3.2	900	10.24	96
65	7.5	4225	56.25	487.5
110	8.4	12100	70.56	924
140	15.1	19600	228.01	2114
185	16.5	34225	272.25	3052.5
Σx = 530	Σy = 50.7	Σx^2 = 71050	Σy^2 = 637.31	Σxy = 6674

$$S_{xx} = \Sigma x^2 - \frac{(\Sigma x)^2}{n}$$

$$S_{xx} = 71050 - \frac{530^2}{5} = 14870$$

$$S_{yy} = \Sigma y^2 - \frac{(\Sigma y)^2}{n}$$

$$S_{yy} = 637.31 - \frac{50.7^2}{5} = 123.212$$

$$S_{xy} = \Sigma xy - \frac{(\Sigma x)(\Sigma y)}{n}$$

$$S_{xy} = 6674 - \frac{530 \times 50.7}{5} = 1299.8$$

$y = ax + b$, where

$$a = \frac{S_{xy}}{S_{xx}} = \frac{1299.8}{14870} = 0.0874 \text{ and}$$

$$b = \bar{y} - a\bar{x} = \frac{50.7}{5} - 0.0874 \times \frac{530}{5}$$

$$= 0.876$$

b The gradient indicates that a car travelling one additional mile uses 0.0874 litres of fuel

c $y = 0.876 + 0.0874 \times 160 = 14.9$ litres

d Not reliable as 5 is outside the domain of the original data

8 a

x	y	x^2	y^2	xy
1	6	1	36	6
1.5	7	2.25	49	10.5
2	10	4	100	20
2.5	15	6.25	225	37.5
3	9	9	81	27
3.5	17	12.25	289	59.5
4	20	16	400	80
4.5	18	20.25	324	81
Σx = 22	Σy = 102	Σx^2 = 71	Σy^2 = 1504	Σxy = 321.5

$$S_{xx} = \Sigma x^2 - \frac{(\Sigma x)^2}{n}$$

$$S_{xx} = 71 - \frac{22^2}{8} = 10.5$$

$$S_{yy} = \Sigma y^2 - \frac{(\Sigma y)^2}{n}$$

$$S_{yy} = 1504 - \frac{102^2}{8} = 203.5$$

$$S_{xy} = \Sigma xy - \frac{(\Sigma x)(\Sigma y)}{n}$$

$$S_{xy} = 321.5 - \frac{22 \times 102}{8} = 41$$

$y = ax + b$, where

$a = \frac{S_{xy}}{S_{xx}} = \frac{41}{10.5} = 3.90$ and

$b = \bar{y} - a\bar{x} = \frac{102}{8} - 3.90 \times \frac{22}{8} = 2.01$

b An increase in one gram of hormone leads to just under 4 extra flowers

c A plant with no growth hormone will produce 2 flowers

d $y = 2.01 + 3.905 \times 1.75 = 8.84$

e $12 = 2.01 + 3.905x$

$3.905x = 12 - 2.01 \Rightarrow x = \frac{9.99}{3.905} = 2.56$ g

f Not appropriate as 1000 is far outside the domain of the data provided

9 a

x	y	x^2	y^2	xy
100	204	10000	41616	20400
200	257	40000	66049	51400
300	292	90000	85264	87600
400	315	160000	99225	126000
500	330	250000	108900	165000
600	355	360000	126025	213000
700	370	490000	136900	259000
$\Sigma x = 2800$	$\Sigma y = 2123$	$\Sigma x^2 = 1400000$	$\Sigma y^2 = 663979$	$\Sigma xy = 922400$

$S_{xx} = \Sigma x^2 - \frac{(\Sigma x)^2}{n}$

$S_{xx} = 1400000 - \frac{2800^2}{7} = 280000$

$S_{yy} = \Sigma y^2 - \frac{(\Sigma y)^2}{n}$

$S_{yy} = 663979 - \frac{2123^2}{7} = 20103.4$

$S_{xy} = \Sigma xy - \frac{(\Sigma x)(\Sigma y)}{n}$

$S_{xy} = 922400 - \frac{2800 \times 2123}{7} = 73200$

$y = ax + b$, where

$a = \frac{S_{xy}}{S_{xx}} = \frac{73200}{280000} = 0.261$ and

$b = \bar{y} - a\bar{x} = \frac{2123}{7} - 0.261 \times \frac{2800}{7} = 199$

b Each additional gram increases the length of the spring by 0.261 mm

c The spring was 199 mm long before any weight was added

d $y = 199 + 0.261 \times 550 = 343$ mm

e 2 kg is outside the domain of the data, so extrapolation is unreliable

f $x = ay + b$, where

$a = \frac{S_{xy}}{S_{yy}} = \frac{73200}{20103.4} = 3.641$ and

$b = \bar{x} - a\bar{y} = \frac{2800}{7} - 3.641 \times \frac{2123}{7} = -704.263$

so $x = 3.641y - 704.263$,

$x = 3.641 \times 300 - 704.263 = 388$ g

10 a

x	y	x^2	y^2	xy
15	26	225	676	390
25	30	625	900	750
35	25	1225	625	875
45	26	2025	676	1170
55	20	3025	400	1100
65	14	4225	196	910
$\Sigma x = 240$	$\Sigma y = 141$	$\Sigma x^2 = 11350$	$\Sigma y^2 = 3473$	$\Sigma xy = 5195$

$S_{xx} = \Sigma x^2 - \frac{(\Sigma x)^2}{n}$

$S_{xx} = 11350 - \frac{240^2}{6} = 1750$

$S_{yy} = \Sigma y^2 - \frac{(\Sigma y)^2}{n}$

$S_{yy} = 3473 - \frac{141^2}{6} = 159.5$

$S_{xy} = \Sigma xy - \frac{(\Sigma x)(\Sigma y)}{n}$

$S_{xy} = 5195 - \frac{240 \times 141}{6} = -445$

$y = ax + b$, where

$a = \frac{S_{xy}}{S_{xx}} = \frac{-445}{1750} = -0.254$ and

$b = \bar{y} - a\bar{x} = \frac{141}{6} + 0.254 \times \frac{240}{6} = 33.7$,

$y = 33.7 - 0.254x$

b $y = 33.7 - 0.254 \times 50$

$= 21$ decimal places

c $r = \frac{S_{xy}}{\sqrt{(S_{xx}S_{yy})}}$

$r = \frac{-445}{\sqrt{1750 \times 159.5}} = -0.842$

d There is a strong, negative correlation

11 a $0.51 \times 120 + 7.5 = 68.7$ M1A1

b The line of best fit goes through $(\bar{x}, \bar{y})$ R1

$\bar{y} = 0.51 \times 100 + 7.5 = 58.5$ A1

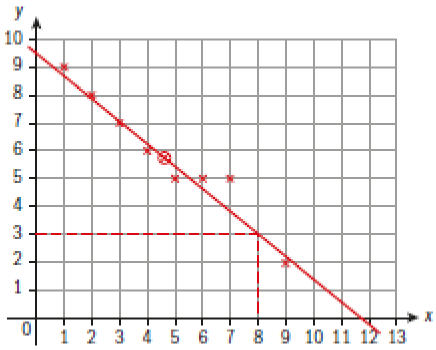
c Strong, positive A1A1

d x on y A1

12 i perfect positive A1
ii strong negative A1
iii weak positive A1
iv weak negative A1
v zero A1

13 a $r = 0.979$ (3sf) A2
b Strong, positive A1A1
c i $y = 1.23x - 21.3$ A1A1
ii $x = 0.776y + 20.8$ A1A1
d $1.23 \times 105 - 21.3 = 110$ A1
e $0.776 \times 95 + 20.8 = 95$ A1
f It is extrapolation R1

14 a



(scales: A1; 3 points plotted correctly: A2; all points plotted correctly: award a further A1)

b strong, negative A1A1
c i $\bar{x} = 4.625$
ii $\bar{y} = 5.875$
iii see above A2A2A1
d see above M1
line passes through the mean A1
e 3.2 see above for lines drawn on A1A1

15 a $100 = 70m + c$

$140 = 100m + c$

$40 = 30m$

$m = \frac{4}{3}$

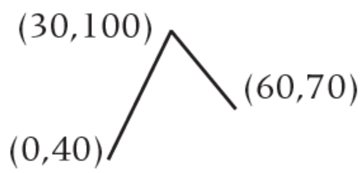
$c = \frac{20}{3}$ (M1)A1A1

b Positive A1
c Line goes through $(\bar{x}, \bar{y})$ (R1)

$\bar{y} = \frac{4}{3}90 + 6\frac{2}{3} = 126\frac{2}{3}$ (M1)A1

d Estimate is $\frac{4}{3}60 + 6\frac{2}{3} = 86\frac{2}{3}$ (M1)A1

16 a 40℃ A1
b 70℃ A1
c 100℃ A1
d i



A1

ii $T \geq 80$

$40 + 2t = 80 \Rightarrow t = 20$

$130 - t = 80 \Rightarrow t = 50$ M1

Interval is $20 \leq t \leq 50$. A1A1

17 a

x	13	14	15	16	16	17	18	18	19	19
y	2	0	3	1	4	1	1	2	1	2

A3 (A2 for 5 A1 for 3)

b $r = -0.0695(3sf)$ A2

c Very weak (negative) correlation so line of best fit is meaningless R1

25-year-old would be extrapolation R1

18 i Gradient $m = \frac{0.6}{3} = 0.2$ M1A1
ii $l = 0.6$ A1
iii $k = 3$ A1
iv $a = 5$ A1
v $b = 0.6$ A1
vi Gradient $p = \frac{0.9 - 0.6}{8 - 5} = 0.1$ M1A1
vii $0.6 = 0.1 \times 5 + q \Rightarrow q = 0.1$ M1A1
viii $r = 8$ A1

19 a i 0.849 (3sf) A2
ii strong, positive A1A1
iii $y = 0.937x + 0.242$ A1A1
b i 0.267 (3sf) A2
ii weak, positive A1A1
iii the r value is too small for this to be particularly meaningful R1

20 a i no change
$r = 0.87$ A1
ii no change
15 A1
iii The scatter diagram has just been moved down by 4 and to the right by 5. R1
iv Strong, positive A1A1

b i no change
$r = 0.87$ A1

ii $2 \times 15 = 30$ A1

iii the scatter diagram has been stretched vertically. R1

c **i** $r = -0.87$ A1

ii $\frac{15}{-3} = -5$ A1

iii The scatter diagram has been stretched horizontally and reflected in the y-axis. R1R1

iv Strong, negative A1A1

8 Quantifying randomness: probability

Skills Check

1 a $1-\frac{3}{7}=\frac{7}{7}-\frac{3}{7}=\frac{4}{7}$

b $\frac{2}{5}+\frac{5}{7}=\frac{14}{35}+\frac{25}{35}=\frac{39}{35}$

c $\frac{2}{5}\times\frac{2}{3}=\frac{2\times 2}{5\times 3}=\frac{4}{15}$

d $1-\left(\frac{1}{7}\times\frac{3}{8}\right)=1-\frac{3}{56}=\frac{53}{56}$

e $\dfrac{\frac{3}{20}}{\frac{7}{20}}=\frac{3\div 20}{7\div 20}=\frac{3}{7}$

2 a $1-0.375=0.625$

b $0.65+0.05=0.7$

c $0.7\times 0.6=\frac{7\times 6}{10^2}=\frac{42}{100}=0.42$

d $0.25\times 0.64=0.64\div 4=0.16$

e $0.5\times 30=30\div 2=15$

f $0.22\times 0.22=\frac{22^2}{100^2}=\frac{484}{10000}=0.0484$

Exercise 8A

1 $P(\text{odd})=\frac{n(\{1,3,5,7,9\})}{n(\{1,2,3,4,5,6,7,8,9,10\})}$

$=\frac{5}{10}=\frac{1}{2}$

2 $P(\text{defective})=\frac{30}{150}=\frac{1}{5}$

3 $P(\text{chorus})=\frac{20}{20+10+5}=\frac{20}{35}=\frac{4}{7}$

4 a $P(\text{even})=\frac{n(\{2,4,6,8\})}{n(\{1,2,3,4,5,6,7,8\})}=\frac{4}{8}=\frac{1}{2}$

b $P(\text{multiple of 3})=\frac{n(\{3,6\})}{n(\{1,2,3,4,5,6,7,8\})}$

$=\frac{2}{8}=\frac{1}{4}$

c $P(\text{multiple of 4})=\frac{n(\{4,8\})}{n(\{1,2,3,4,5,6,7,8\})}$

$=\frac{2}{8}=\frac{1}{4}$

d $P(\text{not a multiple of 4})$

$=1-P(\text{multiple of 4})=1-\frac{1}{4}=\frac{3}{4}$

e $P(\text{less than 4})=\frac{n(\{1,2,3\})}{n(\{1,2,3,4,5,6,7,8\})}$

$=\frac{3}{8}$

f As there is no 9 on an 8 sided dice,
$P(9)=0$

5 a $P(C)=\frac{n(\{C\})}{n(\{S,T,A,T,I,S,T,I,C,S\})}=\frac{1}{10}$

b As there is no P in the word "STATISTICS", $P(P)=0$

c $P(\text{vowel})=\frac{n(\{A,I,I\})}{n(\{S,T,A,T,I,S,T,I,C,S\})}$

$=\frac{3}{10}$

6 a Every other number is even, so

$P(\text{even})=\frac{1}{2}$

b $P(\text{contains digit 1})$

$=\frac{n(\{1,10-19,21,31,41\})}{50}=\frac{14}{50}=\frac{7}{25}$

7 Let x be the number of seats on a minibus, then

$P(\text{coach})=\frac{3x}{x+x+x+x+3x}=\frac{3x}{7x}=\frac{3}{7}$

8 Using $P(\text{green})=2P(\text{yellow})$ and

$1=P(\text{red})+P(\text{yellow})+P(\text{blue})+P(\text{green})$,

we see that

$1=P(\text{red})+P(\text{yellow})+P(\text{blue})+P(\text{green})$

$1=0.4+P(\text{yellow})+0.3+P(\text{green})$

$1=0.7+P(\text{yellow})+P(\text{green})$

$1-0.7=P(\text{yellow})+2P(\text{yellow})$

$0.3=3P(\text{yellow})$

$\frac{0.3}{3}=P(\text{yellow})$

$P(\text{yellow})=0.1$

$P(\text{green})=2P(\text{yellow})=0.2$

9 Number of people who buy raffle tickets $=\frac{360}{2}=180$, of these half bought 2 tickets and the other half bought one, so there were $2\times\frac{180}{2}+180=360$ raffle tickets sold. Therefore

$P(\text{win})=\frac{1}{360}$

Exercise 8B

1 a i $P(\text{age 15})=0.18$

ii $P(\text{age 16 or higher})$

$=P(\text{age 16})+P(\text{age 17})+P(\text{age 18})$

$=0.22+0.27+0.13=0.62$

b Number of 15 year old students

$=1200\times P(\text{age 15})=1200\times 0.18=216$

2 **a** The relative frequency of getting a 1 is

$\dfrac{\text{frequency of 1}}{\text{total spins}} = \dfrac{27}{100} = 0.27$

b The spinner is probably not fair because the relative frequencies are not close to each other, a 1 occurred nearly 4 times more than a 6

c Estimated number of 4s $= 3000 \times \dfrac{15}{100} = 450$

3 **a** 10 of each

b

Relative frequency	0.1	0.1	0.15	0.138	0.138	0.15	0.138	0.0875

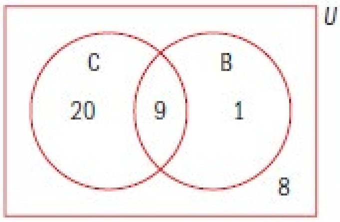
c

Relative frequency	0.0925	0.1225	0.1375	0.125	0.14	0.145	0.1075	0.13

d There is a big difference between relative frequency of getting a 1 and getting a 6. This suggests that the dice is not fair.

Exercise 8C

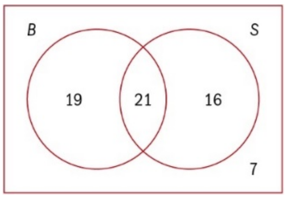
1 **a**



b From the Venn diagram,

$P(\text{neither}) = \dfrac{8}{38} = \dfrac{4}{19}$

2 **a**



b From the Venn diagram,
$19 + 21 + 16 + 7 = 63$

c **i** $P(\text{badminton}) = \dfrac{40}{63}$

ii $P(\text{both}) = \dfrac{21}{63} = \dfrac{1}{3}$

iii $P(\text{neither}) = \dfrac{7}{63} = \dfrac{1}{9}$

iv $P(\text{at least one}) = 1 - P(\text{neither})$

$= 1 - \dfrac{1}{9} = \dfrac{8}{9}$

3 Let A = gave a card and B = gave a present

a $P(\text{card or present}) = \dfrac{31 + 40 - 25}{50} = \dfrac{23}{25}$

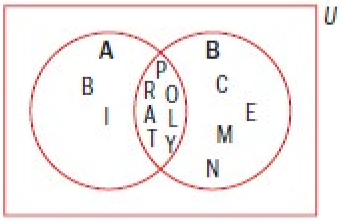
b $P(\text{card but no present}) = \dfrac{31 - 25}{50} = \dfrac{3}{25}$

c $P(\text{neither card nor present})$

$= 1 - P(\text{card or present}) = 1 - \dfrac{23}{25} = \dfrac{2}{25}$

4 $A = \{P, R, O, B, A, I, L, T, Y\}$ and
$B = \{C, O, M, P, L, E, N, T, A, R, Y\}$

a



b $A \cap B = \{P, R, O, A, L, T, Y\}$

c $A \cup B = \{P, R, O, B, A, I, L, T, Y, C, M, E, N\}$

5 **a** $A \cap B = \{6\}$

b $A \cup B = \{2, 3, 4, 6, 8, 9, 10\}$

c $A' = \{1, 3, 5, 7, 9\}$

d $A' \cap B = \{1, 3, 5, 7, 9\} \cap \{3, 6, 9\} = \{3, 9\}$

e $A \cup B'$

$= \{2, 4, 6, 8, 10\} \cup \{1, 2, 4, 5, 7, 8, 10\}$

$= \{1, 2, 4, 5, 6, 7, 8, 10\}$

f $A' \cup B'$

$= \{1, 3, 5, 7, 9\} \cup \{1, 2, 4, 5, 7, 8, 10\}$

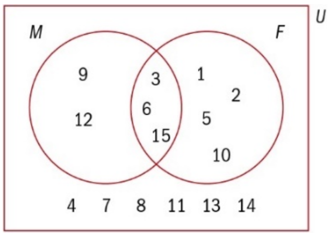
$= \{1, 2, 3, 4, 5, 7, 8, 9, 10\}$

6 $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15\}$

a **i** $M = \{3, 6, 9, 12, 15\}$

ii $F = \{1, 2, 3, 5, 6, 10, 15\}$

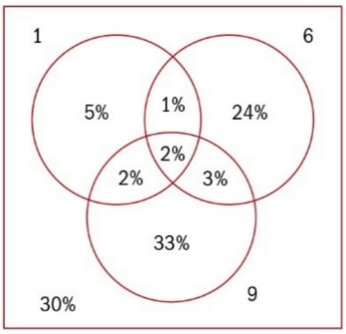
b



c **i** $P(M \cap F) = \dfrac{3}{15} = \dfrac{1}{5}$

ii $P((M \cup F)') = \dfrac{6}{15} = \dfrac{2}{5}$

7 **a**



b **i** $P(\text{only 9 pm}) = 33\%$

ii $P(\text{only 6 pm}) = 24\%$

iii $P(\text{no news}) = 30\%$

Exercise 8D

1 **a** $P(\text{prime}) = \dfrac{n(\{2,3,5,7\})}{10} = \dfrac{4}{10} = \dfrac{2}{5}$

b $P(\text{prime or multiple of 3})$

$= \dfrac{n(\{2,3,5,7\}) + n(\{3,6,9\}) - n(\{3\})}{10}$

$= \dfrac{4+3-1}{10} = \dfrac{6}{10} = \dfrac{3}{5}$

c $P(\text{multiple of 3 or 4})$

$= \dfrac{n(\{3,6,9\}) + n(\{4,8\})}{n(\{10\})}$

$= \dfrac{3+2}{10} = \dfrac{5}{10} = \dfrac{1}{2}$

2 $P(\text{camera owner or female})$

$= \dfrac{n(\text{cam}) + n(\text{fem}) - n(\text{fem and cam})}{n(U)}$

$= \dfrac{30+25-18}{55} = \dfrac{37}{55}$

3 Let $A = \{M,A,T,H,E,I,C,S\}$ and $B = \{T,R,I,G,O,N,M,E,Y\}$

a $P(A) = \dfrac{n(A)}{n(U)} = \dfrac{8}{26} = \dfrac{4}{13}$

b $P(B) = \dfrac{n(B)}{n(U)} = \dfrac{9}{26}$

c $P(A \cap B) = \dfrac{n(\{E,I,M,T\})}{n(U)} = \dfrac{4}{26} = \dfrac{2}{13}$

d $P(A \cup B) = \dfrac{n(A) + n(B) - n(A \cap B)}{n(U)}$

$= \dfrac{8+9-4}{26} = \dfrac{13}{26} = \dfrac{1}{2}$

4 **a** $P(\text{fiction or non-fiction})$

$= P(\text{fiction}) + P(\text{non-fiction}) - P(\text{both})$
$= 0.4 + 0.3 - 0.2 = 0.5$

b $P(\text{no book})$

$= 1 - P(\text{fiction or non-fiction})$
$= 1 - 0.5 = 0.5$

5 **a** $P(X \cup Y) = P(X) + P(Y) - P(X \cap Y)$

$= \dfrac{1}{4} + \dfrac{1}{8} - \dfrac{1}{8} = \dfrac{1}{4}$

b $P(X \cup Y)' = 1 - P(X \cup Y) = 1 - \dfrac{1}{4} = \dfrac{3}{4}$

6 **a** $P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$= 0.2 + 0.4 - 0.5 = 0.1$

b $P(A' \cup B) = P(A') + P(B) - P(A' \cap B)$

$= 1 - P(A) + P(B) - (P(B) - P(A \cap B))$

$= 1 - 0.2 + 0.4 - (0.4 - 0.1) = 0.9$

7 **a** $3P(A \cap B) = P(A \cup B)$

$= P(A) + P(B) - P(A \cap B)$

$= \dfrac{3}{16} + \dfrac{3}{8} - P(A \cap B)$

so $4P(A \cap B) = \dfrac{9}{16}$

$\Rightarrow P(A \cap B) = \dfrac{9}{64} \Rightarrow P(A \cup B) = \dfrac{27}{64}$

b $P(A \cup B)' = 1 - P(A \cup B) = 1 - \dfrac{27}{64} = \dfrac{37}{64}$

c $P(A \cap B') = P(A) - P(A \cap B)$

$= \dfrac{3}{16} - \dfrac{9}{64} = \dfrac{3}{64}$

Exercise 8E

1 **a** No **b** Yes **c** No **d** Yes

e No **f** No **g** No

2 $P(N \cap M) = P(N) + P(M) - P(N \cup M)$

$= \dfrac{1}{5} + \dfrac{1}{10} - \dfrac{3}{10} = 0$

so N and M are mutually exclusive

3 $P(A \cap B) = P(A \cap C) = P(B \cap C) = 0$ because only one school can win

a $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \dfrac{1}{3} + \dfrac{1}{4} - 0 = \dfrac{7}{12}$

b $P(A \cup B \cup C) = P(A) + P(B) + P(C)$

$= \dfrac{1}{3} + \dfrac{1}{4} + \dfrac{1}{5} = \dfrac{47}{60}$

c Yes, because the probability of A, B or C winning is not equal to 1.

Exercise 8F

1 $U = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$

a $P(\text{more heads than tails})$

$= \dfrac{n(\{HHH, HHT, HTH, THH\})}{n(U)} = \dfrac{4}{8} = \dfrac{1}{2}$

b $P(\text{at least two heads consecutively})$

$= \dfrac{n(\{HHH, HHT, THH\})}{n(U)} = \dfrac{3}{8}$

c $P(\text{heads and tails alternately})$

$= \dfrac{n(\{HTH, THT\})}{n(U)} = \dfrac{2}{8} = \dfrac{1}{4}$

2 **a**

	1	2	3	4
1	1, 1	1, 2	1, 3	1, 4
2	2, 1	2, 2	2, 3	2, 4
3	3, 1	3, 2	3, 3	3, 4
4	4, 1	4, 2	4, 3	4, 4

b **i** $P(\text{red is higher than blue})$

$= \frac{n(\{(1,2),(1,3),(1,4),(2,3),(2,4),(3,4)\})}{n(U)}$

$= \frac{6}{16} = \frac{3}{8}$

ii P(difference between numbers is 1)

$= \frac{n(\{(1,2),(2,3),(3,4),(2,1),(3,2),(4,3)\})}{n(U)}$

$= \frac{6}{16} = \frac{3}{8}$

iii P(red is odd and blue is even)

$= \frac{n(\{(1,2),(1,4),(3,2),(3,4)\})}{n(U)}$

$= \frac{4}{16} = \frac{1}{4}$

iv P(sum is prime)

$= \frac{n(\{(1,1),(1,2),(1,4),(2,1),(2,3),(3,2),(3,4),(4,1),(4,3)\})}{n(U)}$

$= \frac{9}{16}$

3 a

	1	2	3
2	2, 1	2, 2	2, 3
3	3, 1	3, 2	3, 3
4	4, 1	4, 2	4, 3
5	5, 1	5, 2	5, 3

b i P(cards have same number)

$= \frac{n(\{(2,2),(3,3)\})}{n(U)} = \frac{2}{12} = \frac{1}{6}$

ii P(largest number is 3)

$= \frac{n(\{(2,3),(3,1),(3,2),(3,3)\})}{n(U)}$

$= \frac{4}{16} = \frac{1}{4}$

iii P(sum is less than 7)

$\frac{n(\{(2,1),(2,2),(2,3),(3,1),(3,2),(3,3),(4,1),(4,2),(5,1)\})}{n(U)}$

$= \frac{9}{12} = \frac{3}{4}$

iv P(product is at least 8)

$= \frac{n(\{(3,3),(4,2),(4,3),(5,2),(5,3)\})}{n(U)}$

$= \frac{5}{12}$

v P(at least one even number)

$= \frac{n(\{(2,1),(2,2),(2,3),(3,2),(4,1),(4,2),(4,3),(5,2)\})}{n(U)}$

$= \frac{8}{12} = \frac{2}{3}$

4 a P(at start after 2 rolls)

$= \frac{n(\{(1,3),(2,4),(3,1),(4,2),(5,6),(6,5),(5,5),(6,6)\})}{n(U)}$

$= \frac{8}{36} = \frac{2}{9}$

b P(2 meters from start after 2 rolls)

$= \frac{n(\{(1,1),(2,2),(3,3),(4,4)\})}{n(U)} = \frac{4}{36} = \frac{1}{9}$

c To be more than 1 but less than 2 meters away, he must go to a corner

P(between 1 and 2 meters after 2 rolls)

$= \frac{n(\{(1,2),(2,1),(2,3),(3,2),(3,4),(4,3),(4,1),(1,4)\})}{n(U)}$

$= \frac{8}{36} = \frac{2}{9}$

Exercise 8G

1 P(both purple) $= \frac{1}{5} \times \frac{1}{5} = \frac{1}{25}$

2 P(all 3 like pasta) $= \left(\frac{4}{5}\right)^3 = \frac{64}{125}$

3 P(loses both) $= (1 - 0.75) \times (1 - 0.85)$

$= 0.0375$

4 a $P(B) = P(A \cap B) + P(A \cup B) - P(A)$

$= 0 + 0.4 - 0.2 = 0.2$

$P(B \cap C) = P(B) + P(C) - P(B \cup C)$

$= 0.2 + 0.3 - 0.34$

$= 0.16$

b Not independent as $P(B \cap C) \neq 0$

5 P(head and not 6) $= \frac{1}{2} \times \frac{5}{6} = \frac{5}{12}$

6 P(not hitting with 4 missiles)

$= \left(\frac{1}{9}\right)^4 = \frac{1}{6561}$

7 a $P(E) = 1 - P(E') = 1 - 0.6 = 0.4$

b i Because

$P(E) \times P(F) = 0.24 = P(E \cap F)$

ii Because $P(E \cap F) \neq 0$

c $P(E \cup F') = P(E) + P(F') - P(E \cap F')$

$= P(E) + 1 - P(F) - (P(E) - P(E \cap F))$

$= 0.4 + 1 - 0.6 - 0.4 + 0.24 = 0.64$

8 The only possible way to have a sum of 6 is if all dice show 2. P(sum to 6) $= \left(\frac{2}{6}\right)^3 = \frac{1}{27}$

9 a $P(A \cap B) = P(A) \times P(B) = 0.9 \times 0.3 = 0.27$

b $P(A \cap B') = P(A) - P(A \cap B)$

$= 0.9 - 0.27 = 0.63$

c $P(A \cup B') = P(A) + P(B') - P(A \cap B')$

$= 0.9 + 0.3 - 0.63 = 0.97$

Exercise 8H

1 a $n(\text{both subjects})$

$= n(\text{film}) + n(\text{theatre}) - n(\text{either})$

$= 15 + 20 - (27 - 4) = 12$

b i $P(\text{theatre and not film})$

$= P(\text{theatre}) - P(\text{theatre and film})$

$= \frac{20}{27} - \frac{12}{27} = \frac{8}{27}$

ii $P(\text{theatre or film})$

$= P(\text{theatre}) + P(\text{film})$

$\quad - P(\text{theatre and film})$

$= \frac{20}{27} + \frac{15}{27} - \frac{12}{27} = \frac{23}{27}$

iii $P(\text{theatre} \mid \text{film})$

$= \frac{P(\text{theatre and film})}{P(\text{film})}$

$= \frac{\frac{12}{27}}{\frac{15}{27}} = \frac{12}{15} = \frac{4}{5}$

2 a $P(\text{even} \mid \text{not multiple of } 4)$

$= \frac{P(\text{even and not multiple of } 4)}{P(\text{not multiple of } 4)}$

$= \frac{\frac{n(\{2,6,14\})}{8}}{\frac{n(\{1,2,6,7,11,14,29\})}{8}} = \frac{3}{7}$

b $P(<15 \mid >5)$

$= \frac{P(\text{less than 15 and greater than 5})}{P(\text{greater than 5})}$

$= \frac{\frac{n(\{6,7,11,14\})}{8}}{\frac{n(\{6,7,11,14,24,29\})}{8}} = \frac{4}{6} = \frac{2}{3}$

c $P(\text{less than } 5 \mid \text{less than } 15)$

$= \frac{P(\text{less than 5 and less than 15})}{P(\text{less than 15})}$

$= \frac{\frac{n(\{1,2\})}{8}}{\frac{n(\{1,2,6,7,11,14\})}{8}} = \frac{2}{6} = \frac{1}{3}$

d $P(1 \leftrightarrow 10 \mid 5 \leftrightarrow 25)$

$= \frac{P(1 \leftrightarrow 15 \text{ and } 5 \leftrightarrow 25)}{P(5 \leftrightarrow 25)}$

$= \frac{\frac{n(\{6,7,11,14\})}{8}}{\frac{n(\{6,7,11,14,24\})}{8}} = \frac{4}{5}$

3 a $P(V \cap W) = 0$ because they are mutually exclusive

b $P(V \mid W) = 0$ because they are mutually exclusive

c $P(V \cup W) = P(V) + P(W) - P(V \cap W)$

$= 0.26 + 0.37 - 0 = 0.63$

4 a $P(\text{male and left-handed}) = \frac{5}{50} = \frac{1}{10}$

b $P(\text{right-handed}) = \frac{43}{50}$

c $P(\text{right-handed} \mid \text{female})$

$= \frac{P(\text{right-handed and female})}{P(\text{female})} = \frac{\frac{11}{50}}{\frac{13}{50}} = \frac{11}{13}$

5 $P(J \mid K) = \frac{P(J \cap K)}{P(K)} = \frac{P(J) \times P(K)}{P(K)} = P(J) = 0.3$

6 $P(\text{two boys} \mid \text{one is a boy})$

$= \frac{P(\text{two boys and one is a boy})}{P(\text{one is a boy})}$

$= \frac{\frac{n(\{BB\})}{n(\{BB,BG,GB,GG\})}}{\frac{n(\{BB,BG,GB\})}{n(\{BB,BG,GB,GG\})}} = \frac{1}{3}$

Exercise 8I

1 a $P(\text{three picture cards}) = \frac{12}{52} \times \frac{11}{51} \times \frac{10}{50} = \frac{11}{1105}$

b $P(\text{two picture cards}) = 3 \times \frac{12}{52} \times \frac{11}{51} \times \frac{40}{50}$

$= \frac{132}{1105}$

2 a $P(\text{two broken pens}) = \frac{5}{14} \times \frac{4}{13} = \frac{20}{182} = \frac{10}{91}$

b $P(\text{at least one broken pen})$

$= P(\text{one broken pen}) + P(\text{two broken pens})$

$= 2 \times \frac{9}{14} \times \frac{5}{13} + \frac{10}{91} = \frac{55}{91}$

c $P(\text{girl picks broken pen}) = \frac{1}{4}$

3 a $P(\text{male}) = \frac{3}{10}$

b $P(\text{one male and one female})$

$= 2 \times \frac{3}{10} \times \frac{7}{9} = \frac{7}{15}$

4 a $P(\text{at least one answers correctly})$

$= P(\text{one answers correctly})$

$\quad + P(\text{both answer correctly})$

$= \frac{5}{7} \times \frac{4}{9} + \frac{2}{7} \times \frac{5}{9} + \frac{5}{7} \times \frac{5}{9} = \frac{55}{63}$

b $P(\text{Luca correct} \mid \text{at least one correct})$

$= \frac{P(\text{Luca correct})}{P(\text{at least one correct})} = \frac{\frac{5}{7}}{\frac{55}{63}} = \frac{9}{11}$

c $P(\text{Ian correct} \mid \text{at least one correct})$

$$= \frac{P(\text{Ian correct})}{P(\text{at least one correct})} = \frac{\frac{5}{9}}{\frac{55}{63}} = \frac{7}{11}$$

d $P(\text{two correct} \mid \text{at least one correct})$

$$= \frac{P(\text{two correct})}{P(\text{at least one correct})} = \frac{\frac{5}{7}\times\frac{5}{9}}{\frac{55}{63}} = \frac{5}{11}$$

Chapter review

1 a $P(\text{divisible by 5})$

$$= \frac{n(\{10,15,20,\ldots 85,90,95\})}{n(\{10,11,\ldots,98,99\})}$$

$$= \frac{18}{90} = \frac{1}{5}$$

b $P(\text{divisible by 3})$

$$= \frac{n(\{12,15,18,\ldots 93,96,99\})}{n(\{10,11,\ldots,98,99\})} = \frac{30}{90} = \frac{1}{3}$$

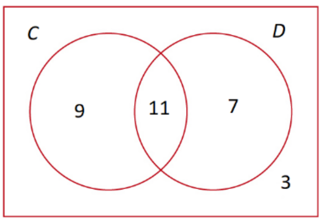
c $P(\text{greater than 50})$

$$= \frac{n(\{51,52,53,\ldots,98,99\})}{n(\{10,11,\ldots,98,99\})} = \frac{49}{90}$$

d $P(\text{a square number})$

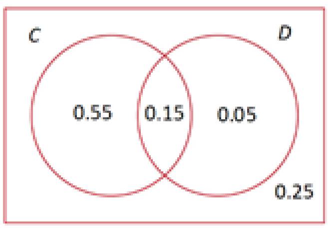
$$= \frac{n(\{16,25,36,49,64,81\})}{n(\{10,11,\ldots,98,99\})} = \frac{6}{90} = \frac{1}{15}$$

2



From Venn diagram $P(\text{Cat and dog}) = \frac{11}{30}$

3 a



From Venn diagram $P(C \cap D') = 0.55$

b $P(C \cap D) = 0.15$

$P(C) \times P(D) = 0.7 \times 0.2 = 0.14$

$P(C \cap D) \neq P(C) \times P(D)$

Therefore C and D are not independent events.

4 a $P(A \cap B) = P(B) \times P(A \mid B)$

$= 0.2 \times 0.1 = 0.02$

b $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.6 + 0.2 - 0.02$

$= 0.78$

c $P(A \cup B) - P(A \cap B)$

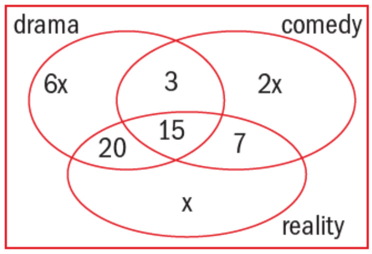
$= 0.78 - 0.02 = 0.76$

d $P(B \mid A) = \frac{P(A \cap B)}{P(A)}$

$$= \frac{0.02}{0.6} = \frac{2}{60} = \frac{1}{30}$$

5 a $6x$

b



c $15 + 3 + 7 + 20 + 6x + 2x + x = 90$

$45 + 9x = 90$

$9x = 45$

$x = 5$

6 a $P(C \cap D) = P(D) \times P(C \mid D)$

$= 0.5 \times 0.6 = 0.3$

b Not mutually exclusive as $P(C \cap D) \neq 0$

c $P(C) \times P(D) = 0.4 \times 0.5 = 0.2$

$P(C \cap D) \neq P(C) \times P(D)$

Therefore C and D are not independent events.

d $P(C \cup D) = P(C) + P(D) - P(C \cap D)$

$= 0.4 + 0.5 - 0.3$

$= 0.6$

e $P(D \mid C) = \frac{P(C \cap D)}{P(C)}$

$$= \frac{0.3}{0.4} = \frac{3}{4} = 0.75$$

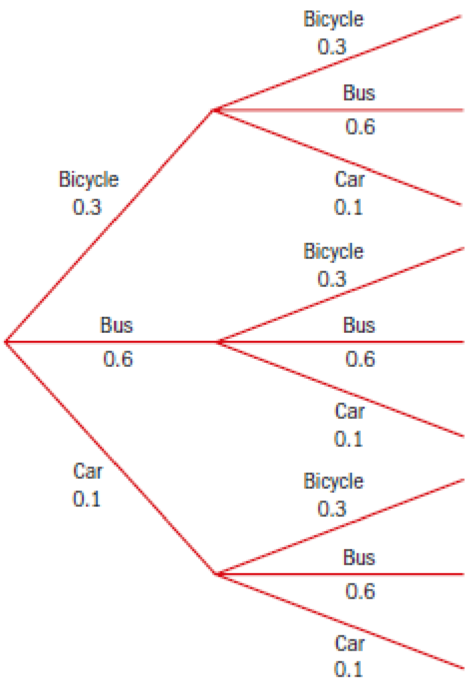
7 a $P(\text{properly}) = \frac{3}{5} \times 0.35 + \frac{2}{5} \times 0.55$

$= 0.21 + 0.22 = 0.43$

b $P(\text{Jill} \mid \text{Properly}') = \frac{P(\text{Jill} \cap \text{Properly}')}{P(\text{Properly}')}$

$$= \frac{\frac{2}{5} \times 0.45}{0.57} = \frac{0.18}{0.57} = 0.316$$

8 a



b i $0.3 \times 0.3 = 0.09$

ii $0.3 \times 0.6 = 0.18$

iii $0.3 \times 0.3 + 0.6 \times 0.6 + 0.1 \times 0.1$

$= 0.09 + 0.36 + 0.01 = 0.46$

c $0.7 \times 0.7 \times 0.7 = 0.343$

d $3 \times 0.1^2 \times 0.6 + 3 \times 0.3^2 \times 0.1$

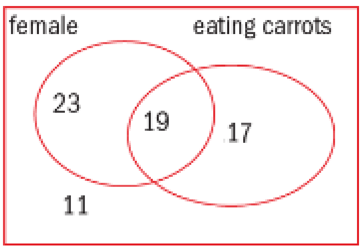
$= 0.018 + 0.027 = 0.045$

9 a $\frac{6}{16} = \frac{3}{8}$

b $\frac{10}{15} = \frac{2}{3}$

c $\frac{5}{15} \times \frac{4}{14} = \frac{1}{3} \times \frac{2}{7} = \frac{2}{21}$

10



Both female and eating carrots = 19.

a $\frac{11}{70}$

b $P(F|C) = \frac{P(F \cap C)}{P(C)} = \frac{19}{36}$

c $P(F) \times P(C) = \frac{42}{70} \times \frac{36}{70}$

$= \frac{54}{175}$

$\neq \frac{19}{70} = P(F \cap C)$

Therefore not independent.

11 a $\frac{1}{6}$ A1

b 2,4 or 6: $\frac{3}{6} = \frac{1}{2}$ A1

c Primes are 2,3,5: $\frac{3}{6} = \frac{1}{2}$ (M1)A1

d 4 or 5: $\frac{2}{6} = \frac{1}{3}$ (M1)A1

e Impossible: 0 A1

12 a $\frac{1}{36}$ A1

b $\frac{6}{36} = \frac{1}{6}$ A1

c $\frac{1}{36}$ A1

d $2 \times \frac{1}{36} = \frac{1}{18}$ A1

e $(1,6),(2,5),(3,4),(4,3),(5,2),(6,1)$

or using a lattice diagram $\frac{6}{36} = \frac{1}{6}$

(M1)A1

f $\frac{1}{6}$ since independent A1

g $P(R5 \cup B5) = P(R5) + P(B5) - P(R5 \cap B5)$

$= \frac{1}{6} + \frac{1}{6} - \frac{1}{36} = \frac{11}{36}$ or using a lattice diagram

(M1)A1

h Considering the list in (e) $\frac{2}{6} = \frac{1}{3}$

or using conditional probability formula

(M1)A1

13 a Independent $\Leftrightarrow P(F \cap R) = P(F) \times P(R)$

R1

$\frac{1}{6} \neq \frac{1}{3} \times \frac{1}{4}$ so not independent A1

b $P(F \cup R) = P(F) + P(R) - P(F \cap R)$ M1

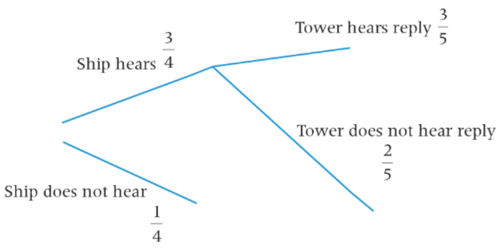
$= \frac{1}{3} + \frac{1}{4} - \frac{1}{6} = \frac{5}{12}$ A1

c P(exactly one team)= $\frac{5}{12} - \frac{1}{6} = \frac{1}{4}$ (M1)A1

Could also use a Venn diagram in (b) and (c)

d $P([F \cap R]|F) = \frac{P(F \cap R)}{P(F)} = \frac{\frac{1}{6}}{\frac{1}{3}} = \frac{1}{2}$ M1A1

14 a



A2

b $\frac{3}{4}\times\frac{3}{5}=\frac{9}{20}$ M1A1

c $1-\frac{9}{20}=\frac{11}{20}$ M1A1

d $P\left(\text{ship not hear}\,|\,\text{tower has no reply}\right)$

$=\frac{P\left(\text{ship not hear}\cap\text{tower has no reply}\right)}{P\left(\text{tower has no reply}\right)}$

$=\frac{\frac{1}{4}}{\frac{11}{20}}=\frac{5}{11}$ M1A1

e $P(A\cap B)=0$ so events are mutually exclusive. R1A1

15 a $30\times\frac{3}{5}=18$ (M1)A1

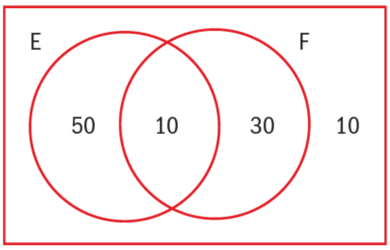
b $50\times\frac{2}{5}=20$ (M1)A1

c $T\times\frac{3}{5}=30\Rightarrow T=50$ (M1)A1

16 a Let x be the number speaking both English and French. $(60-x)+x+(40-x)+10=100$

$\Rightarrow 110-x=100\Rightarrow x=10$ (M1)A1

b



A3 (A1 shape A2 numbers)

c **i** $\frac{50}{100}=\frac{1}{2}$ A1

ii $\frac{90}{100}=\frac{9}{10}$ A1

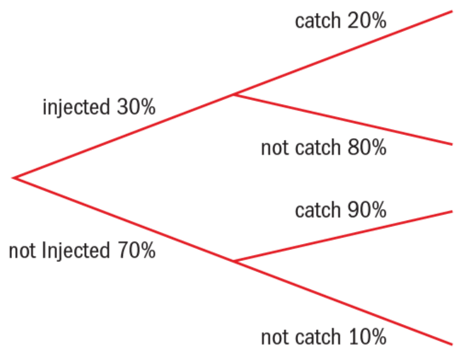
iii $\frac{40}{100}=\frac{2}{5}$ A1

d $P\left(E|F\right)=\frac{P\left(E\cap F\right)}{P\left(F\right)}=\frac{10}{40}=\frac{1}{4}$ M1A1

e If independent then $P\left(E|F\right)=P\left(E\right)$ R1

$\frac{1}{4}\neq\frac{60}{100}$ so not independent A1

17 a



A4 (A2 layout A2 numbers)

b $\frac{70}{100}\times\frac{90}{100}=\frac{63}{100}$ A1

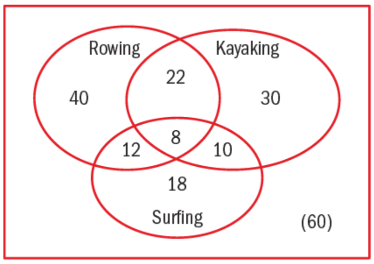
c $\frac{30}{100}\times\frac{80}{100}=\frac{6}{25}$ A1

d $\frac{30}{100}\times\frac{20}{100}+\frac{70}{100}\times\frac{90}{100}=\frac{69}{100}$ M1A1

e $P\left(I'|C\right)=\frac{P\left(I'\cap C\right)}{P\left(C\right)}=\frac{\frac{63}{100}}{\frac{69}{100}}=\frac{21}{23}$ M1A1

f $P\left(I|C'\right)=\frac{P\left(I\cap C'\right)}{P\left(C'\right)}=\frac{\frac{24}{100}}{\frac{31}{100}}=\frac{24}{31}$ M1A1

18 a



A4 (A1 shape, A3, 7 numbers, A2, 4 numbers, A1 2 numbers)

b $200-140=60$ (M1)A1

c **i** $\frac{30}{200}=\frac{3}{20}$ A1

ii $\frac{122}{200}=\frac{61}{100}$ A1

iii $\frac{92}{200}=\frac{23}{50}$ A1

iv $1-\frac{82}{200}=\frac{118}{200}=\frac{59}{100}$ (M1)A1

d $\frac{20}{48}=\frac{5}{12}$ or by using the formula A2

19 a **i** $\frac{5}{8}\times\frac{4}{7}=\frac{5}{14}$ (M1)A1

ii RG or GR

$\frac{5}{8}\times\frac{3}{7}+\frac{3}{8}\times\frac{5}{7}=\frac{15}{28}$ (M1)A1

b **i** $\frac{5}{8}\times\frac{5}{8}=\frac{25}{64}$ (M1)A1

ii RG or GR

$\frac{5}{8}\times\frac{3}{8}+\frac{3}{8}\times\frac{5}{8}=\frac{15}{32}$ (M1)A1

20 **a** **i** $P(A|B)=\frac{P(A\cap B)}{P(B)}$

$\Rightarrow 0.4=\frac{P(A\cap B)}{0.5}\Rightarrow P(A\cap B)=0.2$

M1A1

ii $P(A)=P(A\cap B)+P(A\cap B')$

$=0.2+0.4=0.6$ M1A1

iii $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$=0.6+0.5-0.2=0.9$ M1A1

iv $P(A|B')=\frac{P(A\cap B')}{P(B')}=\frac{0.4}{0.5}=0.8$

M1A1

b $P(A|B)\neq P(A|B')$ so not independent

R1A1