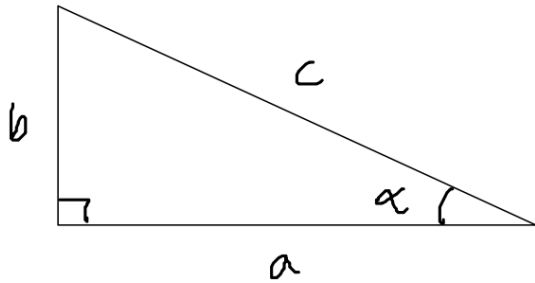
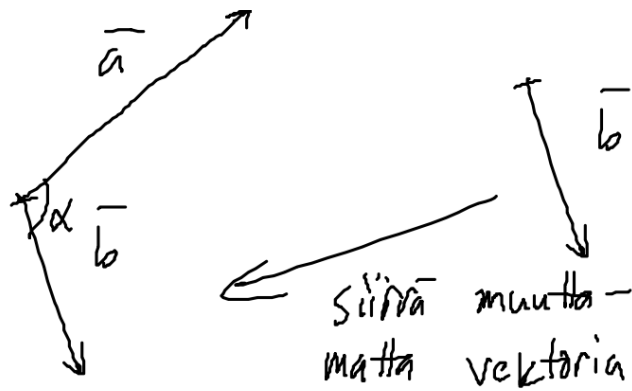


MAB2



- Suorak. kolmio $\rightarrow \cos \alpha = \frac{a}{c} \rightarrow \alpha = \cos^{-1}\left(\frac{a}{c}\right)$
 $\uparrow$
 laskimesta,
 aste-muodi
- α on tällä tapauksessa $0^\circ \dots 90^\circ$,
 mutta $\cos^{-1}(x)$ voi antaa minkä tahansa
 kulman $0^\circ \dots 180^\circ$.

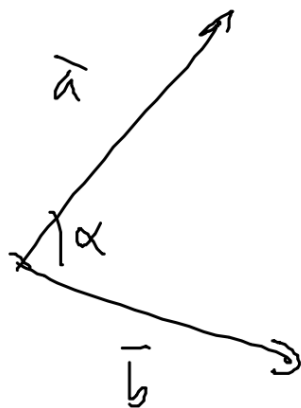
Vektoreiden välisen kulma



$$\cos \alpha = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|}$$

Esim. $\vec{a} = \vec{i} - 2\vec{j} + \vec{k} \rightarrow |\vec{a}| = \sqrt{1^2 + (-2)^2 + 1^2} = \sqrt{6}$
 $\vec{b} = -\frac{1}{2}\vec{i} + \vec{j} + 2\vec{k} \rightarrow |\vec{b}| = \sqrt{\left(-\frac{1}{2}\right)^2 + 1^2 + 2^2} = \sqrt{5,25}$

Laske vektoreiden välinen kulma.

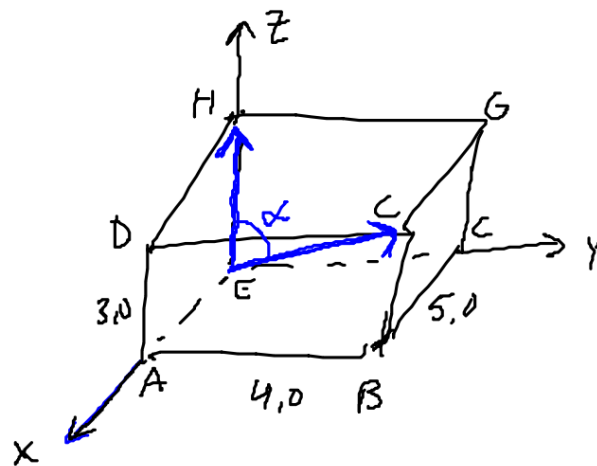


$$\vec{a} \cdot \vec{b} = 1 \cdot \left(-\frac{1}{2}\right) + (-2) \cdot 1 + 1 \cdot 2 = -0,5$$

$$\cos \alpha = \frac{-0,5}{\sqrt{6} \cdot \sqrt{5,25}} = -0,08909$$

$$\alpha = \cos^{-1}(-0,08909) \approx \underline{\underline{95,1^\circ}}$$

142.



(cm)

a) $\angle(\vec{EH}, \vec{EC}) = \alpha$.

$$\vec{EH} = 3\vec{k} \rightarrow |\vec{EH}| = \sqrt{3^2} = \underline{\underline{3}}$$

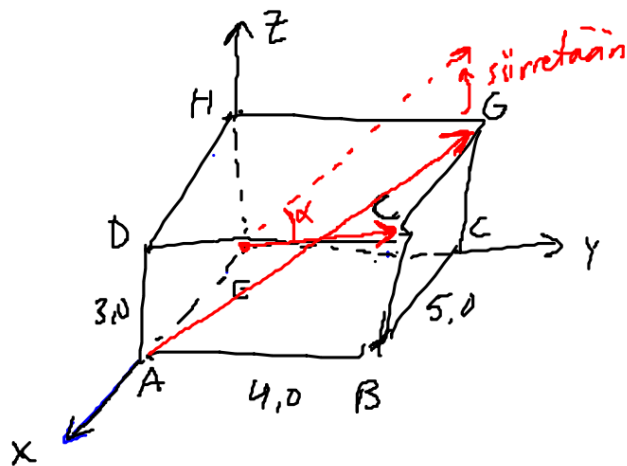
$$\vec{EC} = \vec{EA} + \vec{AB} + \vec{BC} = 5\vec{i} + 4\vec{j} + 3\vec{k}$$

$$\rightarrow |\vec{EC}| = \sqrt{25 + 16 + 9} = \underline{\underline{\sqrt{50}}}$$

$$\vec{EH} \cdot \vec{EC} = 0 \cdot 5 + 0 \cdot 4 + 3 \cdot 3 = \underline{\underline{9}}$$

$$\cos \alpha = \frac{9}{\sqrt{50} \cdot 3} = 0,4243 \rightarrow \alpha = \cos^{-1}(0,4243) \approx 64,9^\circ$$

142.



(cm)

$$b) \quad \angle (\overline{EC}, \overline{AG}) = \alpha$$

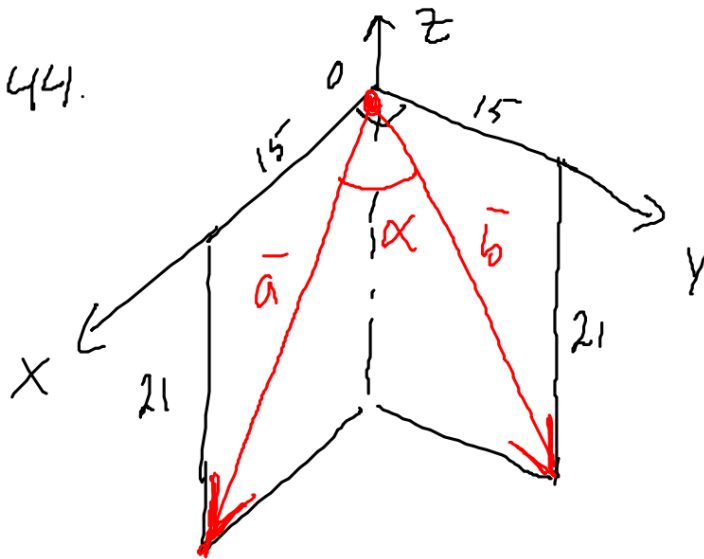
$$\overline{EC} = 5\overline{i} + 4\overline{j} + 3\overline{k} \rightarrow |\overline{EC}| = \sqrt{50}$$

$$\overline{AG} = 3\overline{k} - 5\overline{i} + 4\overline{j} \rightarrow |\overline{AG}| = \sqrt{50}$$

$$\overline{AG} \cdot \overline{EC} = 5 \cdot (-5) + 4 \cdot 4 + 3 \cdot 3 = 0$$

$$\cos \alpha = \frac{0}{\sqrt{50} \cdot \sqrt{50}} = 0 \rightarrow \alpha = \cos^{-1}(0) = \underline{\underline{90^\circ}}$$

144.



(cm)

$$\vec{a} = 15\vec{i} - 21\vec{k}$$

$$\vec{b} = 15\vec{j} - 21\vec{k}$$

$$|\vec{a}| = \sqrt{15^2 + 21^2} = |\vec{b}| = \sqrt{666}$$

$$\vec{a} \cdot \vec{b} = (-21) \cdot (-21) = 441$$

$$\cos \alpha = \frac{441}{\sqrt{666} \cdot \sqrt{666}} = 0,6622$$

$$\alpha = \cos^{-1}(0,6622) \approx 48,5^\circ \sim 50^\circ$$