3 Modelling relationships: linear and quadratic functions

Skills Check

1 **a** $x = -3$ **b** $t = \pm\sqrt{7}$ **c** $a = -\frac{9}{2}$

2 **a** $3m(m-5)$ **b** $(x+6)(x-6)$ **c** $(n+1)(n+7)$ **d** $(x+1)(4x-3)$

e $9x(x+2)$ **f** $(a+1)(2a-5)$ **g** $(3x+2)(4x-1)$ **h** $(4a+7b)(4a-7b)$

Exercise 3A

1 **a** Using the points (-1,0) and (1,-1) on the graph, $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(-1)-0}{1-(-1)} = \frac{-1}{2}$

b Using the points (−5,0) and (0,2) on the graph, $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-0}{0-(-5)} = \frac{2}{5}$

2 **a** $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{11-8}{8-4} = \frac{3}{4}$

b $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(-4)-2}{4-(-2)} = \frac{-6}{6} = -1$

c $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8-1}{7-(-7)} = \frac{7}{14} = \frac{1}{2}$

3 As the line joining the scatter plot (drawn up with t on the x-axis and h on the y-axis) is linear, the gradient can be found by using any two points in the scatterplot: $m = \frac{h_2 - h_1}{t_2 - t_1} = \frac{(4.15)-4.3}{30-20} = \frac{-0.15}{10} = -0.015$. This is the rate of change of the height of the candle, i.e. how fast it is burning down in cm/s.

4 **a** You can use the Pythagorean theorem to find the coordinate of B: as the elevation of B above A is 70m and the direct distance is 350m,

$x_B = \sqrt{350^2 - 70^2} = \sqrt{122500 - 4900}$.

$= \sqrt{117600} \approx 342.93$

Coordinates of B are $(342.93, 100)$.

b $m = \frac{y_2 - y_1}{x_2 - x_1} \approx \frac{100-30}{342.93}$

$= \frac{70}{342.93} \approx 0.20$

c As the gradient is given by $\frac{\text{rise}}{\text{run}}$ itself, grade = gradient × 100% ≈ 20%.

Exercise 3B

1 **a** They are not parallel, as their gradients are not the same, and not perpendicular, as both gradients are positive.

b They are parallel, as $m_1 = -4 = m_2$.

2 As $m * \frac{4}{3} = -1, m = \frac{-3}{4}$. Therefore $\frac{-3}{4} = \frac{5-2}{x-3} = \frac{3}{x-3}$, which is rearranged to $x - 3 = \frac{3*4}{-3} = -4$, yielding $x = -1$.

3 **a** For the first segment, the gradient is given as $m_1 = \frac{320-0}{40-0} = \frac{320}{40} = 8$. The gradient of the second segment $m_2 = \frac{560-320}{60-40} = \frac{240}{20} = 12$.

b This shows that Liam earns 8 dollars per hour regular wage (for the first 40 hours) and 12 dollars per hour worked overtime.

Exercise 3C

1 **a** The gradient is 3, y-intercept is −7.

b The gradient is $-\frac{2}{3}$, y-intercept is 4.

c This could be written as $y = 0x - 2$; thus, the gradient is 0 and the y-intercept is −2.

2 $y = \frac{1}{5}x + 1$ as the gradient is $\frac{1}{5}$ and the y-intercept is 1

3 **a** The gradient is equal to the gradient of $y = 4x - 3$, which is 4, and the y-intercept is -1. Thus $y = 4x - 1$.

b $m = \frac{12}{4} = 3$ and thus $3(1) + a = 10$.
Therefore $a = 10 - 3 = 7$.
Thus $y = 3x + 7$

4 **a** The x-coordinate remains constant so the equation is $x = 8$.

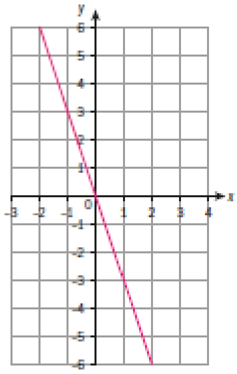
b The y-coordinate remains constant so the equation is $y = -10$

c As horizontal lines are perpendicular to vertical lines, the line is vertical and the equation is $x = 9$.

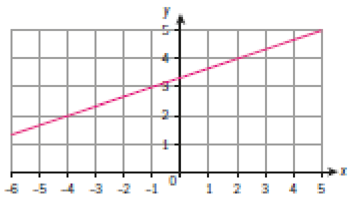
d The lines intersect at the point where $x = -2$ and $y = 7$.

Exercise 3D

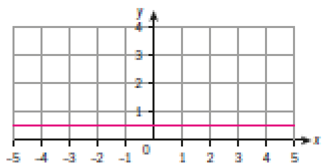
1 a The line goes through $(0,0)$ and through $(1,-3)$.



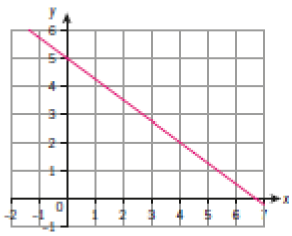
b The point $(-4,2)$ is on the line and so is $(-4+3, 2+1) = (-1,3)$.



c The line is horizontal at $y = \frac{1}{2}$



d The line goes through $(0,5)$ and through $(4,2)$.



2 $y - 6 = -3(x - 2)$

3 a $m = \frac{6}{-2} = -3$

b $y + 4 = -3(x + 3)$ and $y - 2 = -3(x + 5)$ corresponding to the two points given.

c $y + 4 = -3(x + 3)$

$\leftrightarrow y = -3(x + 3) - 4 = -3x - 9 - 4$

$= -3x - 13$

$y - 2 = -3(x + 5)$

$\leftrightarrow y = -3(x + 5) + 2 = -3x - 15 + 2$

$= -3x - 13$

Exercise 3E

1 a $y = \frac{1}{6}x - 3$

$-y + \frac{1}{6}x - 3 = 0$

$-6y + x - 18 = 0$

b $y = -\frac{2}{3}x + 4$

$-y - \frac{2}{3}x + 4 = 0$

$3y + 2x - 12 = 0$

c $y - 2 = -(x + 3)$

$y - 2 = -x - 3$

$y + x - 2 = -3$

$y + x + 1 = 0$

2 a $3x + y - 5 = 0$

$y - 5 = -3x$

$y = -3x + 5$

b $2x - 4y + 8 = 0$

$\frac{1}{2}x - y + 2 = 0$

$y = \frac{1}{2}x + 2$

c $5x + 2y + 7 = 0$

$\frac{5}{2}x + y + \frac{7}{2} = 0$

$y = -\frac{5}{2}x - \frac{7}{2}$

3 a x-intercept:

$x + 2 \times 0 + 6 = 0$

$x + 6 = 0$

$x = -6$

The x-intercept is $(-6,0)$.

y-intercept:

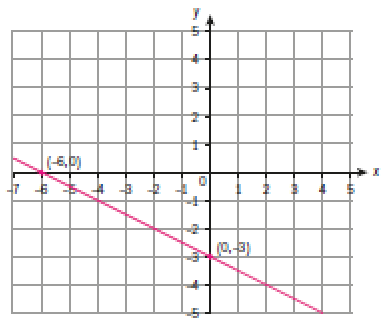
$0 + 2y + 6 = 0$

$2y + 6 = 0$

$2y = -6$

$y = -3$

The y-intercept is $(0,-3)$.



b x-intercept:

$2x - 6 * 0 + 8 = 0$

$2x + 8 = 0$

$2x = -8$

$x = -4$

The x-intercept is $(-4, 0)$.

y-intercept:

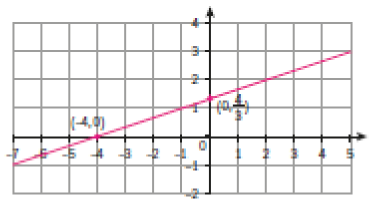
$2 \times 0 - 6y + 8 = 0$

$-6y + 8 = 0$

$6y = 8$

$y = \frac{4}{3}$

The y-intercept is $\left(0, \frac{4}{3}\right)$.



Exercise 3F

1 a $(-2, -5)$ **b** $(0.75, 2.5)$
c $(-3.58, -8.19)$ **d** $(1.18, 1.12)$

2 a 0.9 **b** -5.05

3 \$1666.67

Exercise 3G

1 a $f(3) = -3 + 5 = 2$

b $g(0) = 2 * 0 + 3 = 3$

c $h(6) - g(1) = \left(\frac{1}{3} * 6 - 4\right) - (2 * 1 + 3)$

$= (-2) - 5 = -7$

d $f(2) + g(-1) = (-2 + 5) + (2 * (-1) + 3)$

$= 3 + 1 = 4$

e $(f \circ g)(4) = -g(4) + 5 = -11 + 5 = -6$

f $(h \circ f)(-7) = \frac{1}{3} f(-7) - 4$

$= \frac{1}{3} * 12 - 4 = 4 - 4 = 0$

g $(f \circ g)(x) = -g(x) + 5$

$= -(2x + 3) + 5 = -2x + 2$

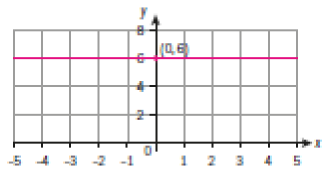
h $(h \circ f)(x) = \frac{1}{3} f(x) - 4 = \frac{1}{3}(-x + 5) - 4$

$= -\frac{1}{3}x + \frac{5}{3} - 4 = -\frac{1}{3}x - \frac{7}{3}$

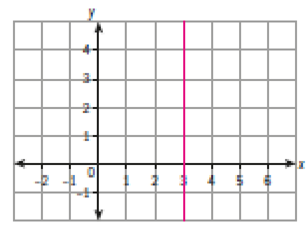
2 a As any real number can be inserted for x and any real number can be obtained as $3x + 8$ for an x, both domain and range are all real numbers.

b Just as above, domain and range are all real numbers.

3 a The line $y = 6$ has range $\{6\}$ as only 6 can be obtained for y.



b No vertical line is a function as the y corresponding to the x-coordinate of the x-intercept is not unique (in fact, any y corresponds to it).



4 a $x = \frac{1}{2}y + 4$

$2x = y + 8$

$f^{-1}(x) = 2x - 8$

b $x = -3y + 9$

$x - 9 = -3y$

$f^{-1}(x) = -\frac{1}{3}x + 3$

Exercise 3H

1 a $x = 4y - 5$

$4y = x + 5$

$y = \frac{1}{4}x + \frac{5}{4}$

$f^{-1}(x) = \frac{1}{4}x + \frac{5}{4}$

b $x = -\frac{1}{6}y + 3$

$-\frac{1}{6}y = x - 3$

$y = -6x + 18$

$f^{-1}(x) = -6x + 18$

c $x = 0.25y + 1.75$

$4x = y + 7$

$y = 4x - 7$

$f^{-1}(x) = 4x - 7$

2 The graph of the inverse function is obtained by mirroring the graph of f at the line $y = x$.

3 a $f(55) = 10 * 55 + 65 = 615$

b $x = 10y + 65$

$10y = x - 65$

$y = 0.1x - 6.5$

$f^{-1}(x) = 0.1x - 6.5$

x here represents the money available in CAD and $f^{-1}(x)$ is the number of t-shirts one can buy with x dollars.

c $y = 0.1 * 5065 - 6.5 = 506.5 - 6.5 = 500$

Exercise 3I

1 a The gradient can be computed from any two points on the line; in this case, a force F of 160 Newtons leads to an extension d of 5 centimetres, while no force (i. e. a force of 0 Newtons) leads to no extension (0 centimetres). Therefore the y-intercept is $(0,0)$ and the gradient is $\frac{5-0}{160-0} = \frac{1}{32}$. This gives the model $d = \frac{1}{32}F$.

b $d = \frac{1}{32} * 370 = 11.5625$ cm.

2 a The gradient is given by $\frac{680-600}{2000-1500} = \frac{80}{500} = 0.16$. As $(1500, 600)$ is on the graph, a point-gradient form of the equation of the line is $y - 600 = 0.16(x - 1500)$. We find the gradient-intercept form:

$y = 0.16(x - 1500) + 600 = 0.16x - 240 + 600 = 0.16x + 3$

.

b The y-intercept represents Frank's basic weekly salary of £360. The gradient shows that Frank's commission is 16% of his sales.

c $y = 0.16 * 900 + 360 = 504$ pounds.

3 a Let y be the total cost in dollars and x the number of months of membership.

For Plan A: $y = 9.99x + 79.99$

For Plan B: $y = 20x$

b We would like to know after how many months the amount paid under each plan is the same (From then onwards, Plan A will be more cost-effective). We therefore solve:

$9.99x + 79.99 = 20x$

$79.99 = 10.01x$

$x = \frac{79.99}{10.01} \approx 7.99$.

Therefore, Plan A is more cost-effective from 8 months onwards.

4 a In the first 40 hours, his pay in pounds is given by $p = \frac{320}{40}h = 8h$. From then on, his pay is given by

$p - 320 = \frac{560-320}{60-40}(h - 40) = 12(h - 40)$

. In gradient-intercept form, this is $p = 12h - 160$.

$$p(h) = \begin{cases} 8h, & 0 \le h \le 40 \\ 12h - 360, & 40 < h \le 60 \end{cases}$$

b i $p = 8 \times 22 = 176$ pounds

ii $p = 12 \times 47 - 160 = 404$ pounds.

5 a $q = -6.5 \times 200 + 3000 = 1700$

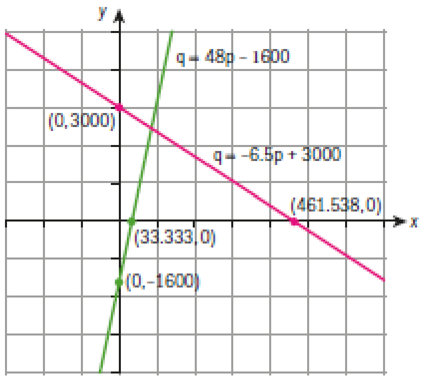
b They will drop by $6.5 \times 20 = 130$ printers a month.

c $2000 = 48p - 1600$

$48p = 3600$

$p = \frac{3600}{48} = 75$ Euro.

d



Use the "solve" function of the GDC.

e Solving $-6.5p + 3000 = 48p - 1600$

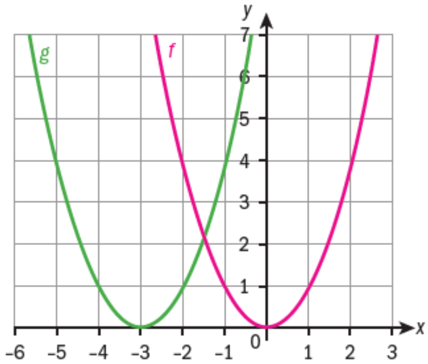
$\Rightarrow p = 84.40..$

So p = €84.40

Then $q = 48 \times 84.40.. - 1600 = 2451$ printers

Exercise 3J

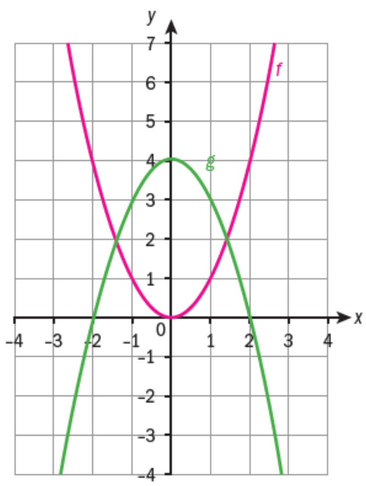
1 a



Axis: x = -3, vertex: (-3, 0)

The graph is translated to the left by 3 units.

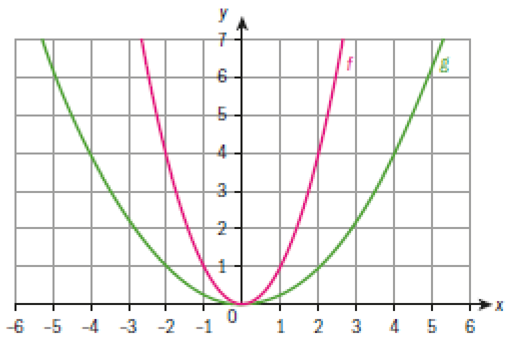
b



Axis: x = 0, vertex: (0, 4)

The graph is reflected about the x-axis and shifted upwards by 4 units.

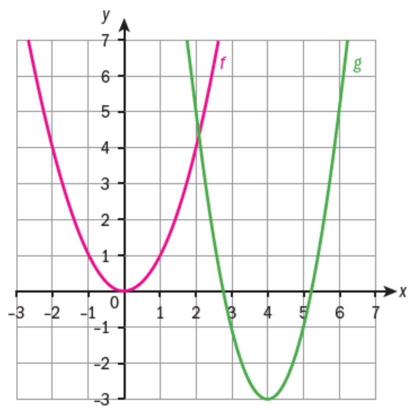
c



Axis: x = 0, vertex: (0, 0)

The graph is compressed vertically with scale factor $\frac{1}{4}$.

d



Axis: x = 4, vertex: (4, -3)

The graph is first translated to the right by 4 units, then stretched vertically with scale factor 2 and finally translated downwards by 3 units.

2 a It is compressed vertically by a scale factor of $\frac{1}{4}$. Thus, the function is given by $g(x) = \frac{1}{4} f(x) = \frac{1}{4} x^2$.

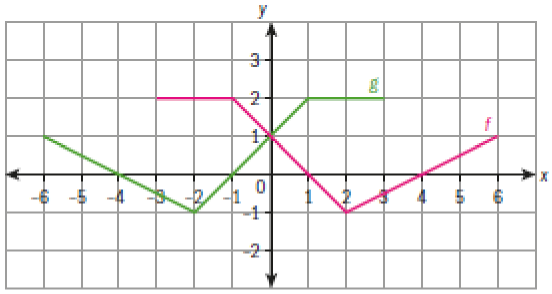
b It is stretched vertically by a scale factor of 2 and reflected along the x-axis. Thus, the function is given by $g(x) = -2f(x) = -2x^2$.

c It is translated to the right by 3 and upwards by 2 units. Thus, the function is given by $g(x) = f(x-3) + 2 = (x-3)^2 + 2$.

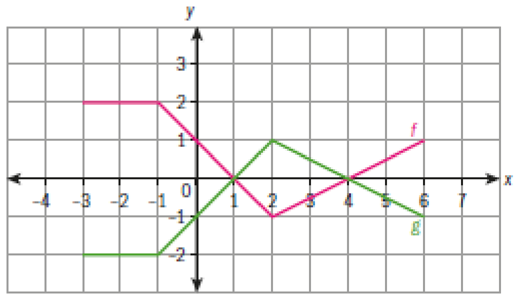
d It is stretched vertically by a scale factor of 1.5, translated to the left by 3 and downwards by 5 units. Thus, the function is given by $g(x) = 1.5f(x+3) - 5 = 1.5(x+3)^2 - 5$.

Exercise 3K

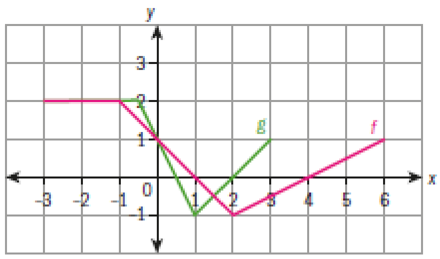
1 a The graph is reflected about the y -axis.



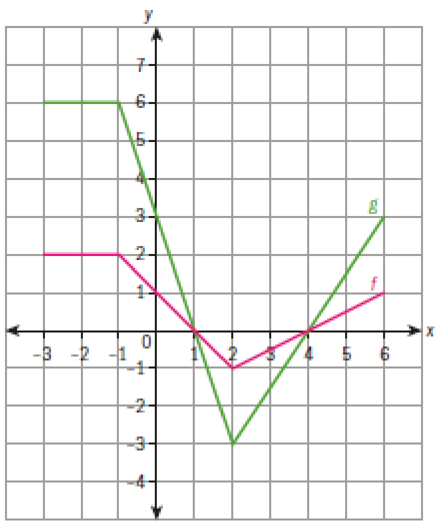
b The graph is reflected about the x -axis.



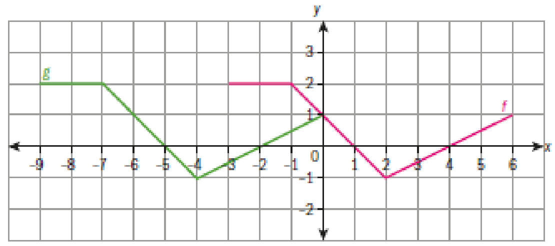
c The graph is compressed horizontally with scale factor $\frac{1}{2}$.



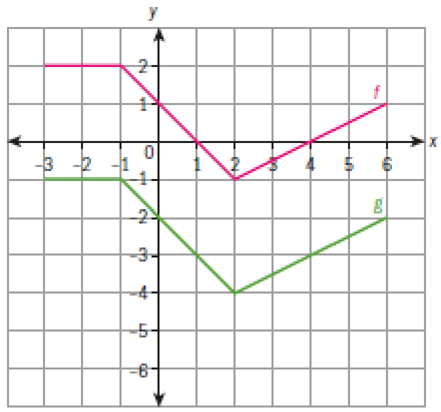
d The graph is stretched vertically with scale factor 3.



e The graph is translated to the left by 6 units.



f The graph is translated downwards by 3 units.



2 a The graph of r is stretched by a scale factor of 2. Thus $r(x) = 2f(x)$.

The graph of s is translated to the right by 3 units and reflected about the x -axis. Thus $s(x) = -f(x-3)$.

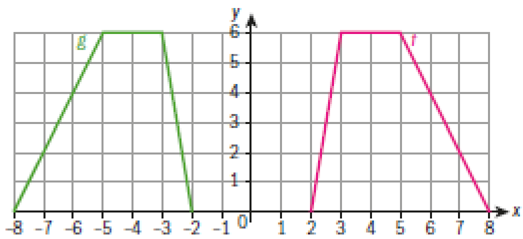
b The graph of r is reflected about the y -axis. Thus $r(x) = f(-x)$.

The graph of s is stretched horizontally by a scale factor of 2 and translated downwards by 4 units. Thus

$$s(x) = f\left(\frac{1}{2}x\right) - 4$$

3 a $0 \le y \le 6$

b It is reflected about the y -axis.



c $2 \le -x \le 8$, which is equivalent to $-8 \le x \le -2$.

d The range of g is the same as the range of f. $0 \le y - c \le 6$ is equivalent to $c \le y \le 6 + c$, so $c = -4$. Thus $h(x) = g(x) - 4$.

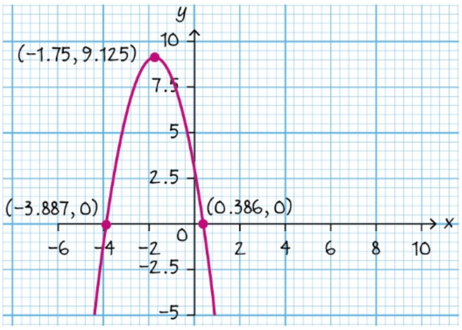
e $h(x) = g(x) - 4 = f(-x) - 4$

Exercise 3L

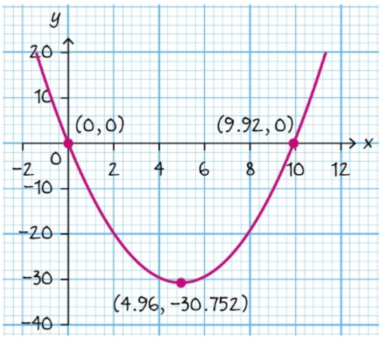
1 x-intercepts: (−2.81, 0), (0.475, 0);
y-intercept: (0, −4);
vertex: (−1.17, −8.08)

2 x-intercepts: none;
y-intercept: (0, −3);
vertex: (0.726, −0.785)

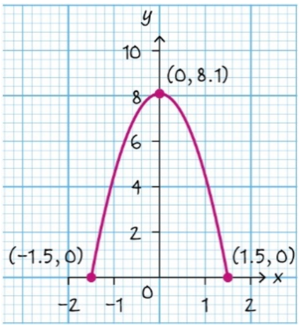
3 Domain: $x \in$ ¡
Range: f(x) ≤ 9.125



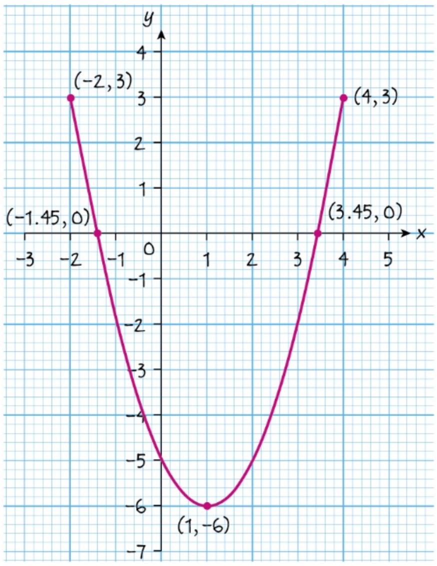
4 Domain: $x \in$ ¡
Range: f(x) ≥ −30.752



5 Range: 0 ≤ f(x) ≤ 8.1



6 Range: $-6 \le f(x) \le 3$



Exercise 3M

1 a $x = 3$ is the axis of symmetry and $(3, 4)$ the coordinates of the vertex.

b $x = 1$ is the axis of symmetry and $(1, -5)$ the coordinates of the vertex.

c $x = -3$ is the axis of symmetry and $(-3, 2)$ the coordinates of the vertex.

d $x = -6$ is the axis of symmetry and $(-6, -5)$ the coordinates of the vertex.

2 a The y-intercept is given by $(0, 5)$, the axis of symmetry is at $x = -\frac{-8}{2} = 4$ and the vertex is at $(4, f(4)) = (4, 16 - 32 + 5) = (4, -11)$.

b The y-intercept is given by $(0, 2)$, the axis of symmetry is at $x = -\frac{-6}{6} = 1$ and the vertex is at $(1, f(1)) = (1, 3 - 6 + 2) = (1, -1)$.

c The y-intercept is given by $(0, -11)$, the axis of symmetry is at $x = -\frac{-8}{-4} = -2$ and the vertex is at $(-2, f(-2)) = (-2, -8 + 16 - 11) = (-2, -3)$.

d The y-intercept is given by $(0, 3)$, the axis of symmetry is at $x = -\frac{6}{4} = -\frac{3}{2}$ and the vertex is at $\left(-\frac{3}{2}, f\left(-\frac{3}{2}\right)\right) = \left(-\frac{3}{2}, \frac{9}{2} - 9 + 3\right) = \left(-\frac{3}{2}, -\frac{3}{2}\right)$
.

3 a The x-intercepts are at $(2,0)$ and $(4,0)$. The axis of symmetry lies at $x=\frac{2+4}{2}=\frac{6}{2}=3$. The vertex is at $(3,f(3))=(3,1*(-1))=(3,-1)$.

b The x-intercepts are at $(-3,0)$ and $(1,0)$. The axis of symmetry lies at $x=\frac{-3+1}{2}=\frac{-2}{2}=-1$. The vertex is at $(-1,f(-1))=(-1,4*2*(-2))=(-1,-16)$.

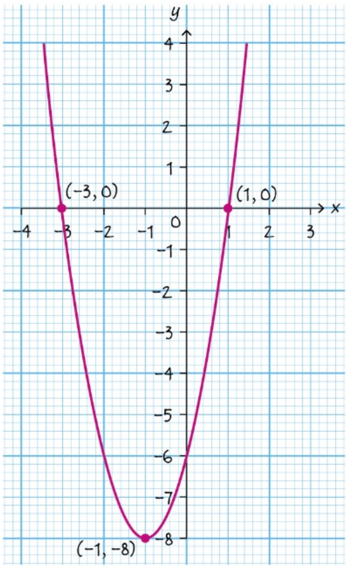
c The x-intercepts are at $(-5,0)$ and $(3,0)$. The axis of symmetry lies at $x=\frac{-5+3}{2}=\frac{-2}{2}=-1$. The vertex is at $(-1,f(-1))=(-1,-(4*(-4))=(-1,16)$.

d The x-intercepts are at $(-3,0)$ and $(-2,0)$. The axis of symmetry lies at $x=\frac{-3-2}{2}=\frac{-5}{2}$. The vertex is at

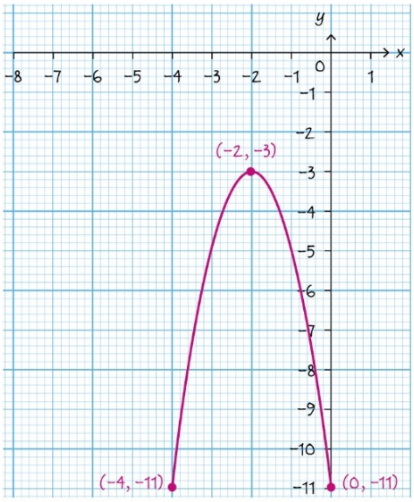
$$\left(\frac{-5}{2},f\left(\frac{-5}{2}\right)\right)=\left(\frac{-5}{2},2*\frac{1}{2}*\left(-\frac{1}{2}\right)\right)$$

$$=\left(\frac{-5}{2},-\frac{1}{2}\right)$$

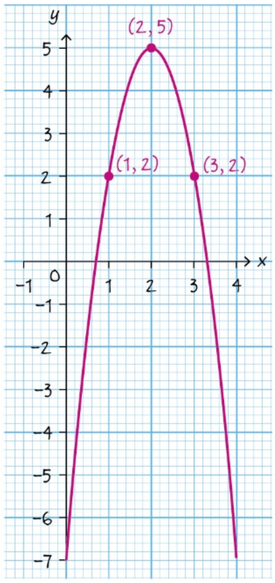
4 a



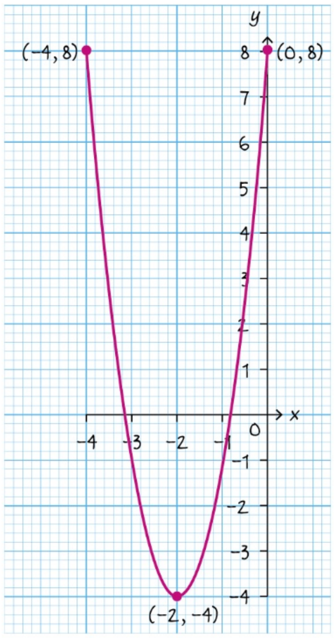
b



c



d



Exercise 3N

1 **a** $f(x)=(x-2)(x+9)$. The x-intercepts are $(2,0)$ and $(-9,0)$ (from the intercept form), and the y-intercept is $(0,-18)$ (from the standard form).

b $f(x)=(3x-5)(x-2)=3(x-2)\left(x-\frac{5}{3}\right)$.

The x-intercepts are $(2,0)$ and $\left(\frac{5}{3},0\right)$ (from the intercept form), and the y-intercept is $(0,10)$ (from the standard form).

c $f(x)=\frac{1}{2}\left(x^2+6x+8\right)=\frac{1}{2}(x+2)(x+4)$.

The x-intercepts are $-(2,0)$ and $(-4,0)$ (from the intercept form), and the y-intercept is $(0,4)$ (from the standard form).

d $f(x)=-(x-4)(4x-2)$

$=-4(x-4)\left(x-\frac{1}{2}\right)$

The x-intercepts are $(4,0)$ and $\left(\frac{1}{2},0\right)$ (from the intercept form), and the y-intercept is $(0,-8)$ (from the standard form).

2 **a** $f(x)=4x^2+16x-20$. The x-intercepts are $(1,0)$ and $(-5,0)$ (from the intercept form), and the y-intercept is $(0,-20)$ (from the standard form).

b $f(x)=-2x^2-16x-14$. The x-intercepts are $(-7,0)$ and $(-1,0)$ (from the intercept form), and the y-intercept is $(0,-14)$ (from the standard form).

3 **a** $f(x)=-3x^2-6x-9$. The vertex is at $(-1,-6)$ (from the vertex form) and the y-intercept is $(0,-9)$.

b $f(x)=\frac{1}{2}x^2-4x+11$. The vertex is at $(4,3)$ (from the vertex form) and the y-intercept is $(0,11)$.

4 **a** $f(x)=(x-4)(x+2)$. Thus

i $a=1$ **ii** $p=4$ **iii** $q=-2$

b **i** The x-intercepts are at $(4,0)$ and $(-2,0)$

ii The y-intercept is at $(0,-8)$

c The axis of symmetry is at $x=\frac{4-2}{2}=1$ Thus the vertex is at $(1,f(1))=(1,(-3)\times 3)=(1,-9)$.

5 **a** **i** The vertex is at $(3,-2)$.

ii The axis of symmetry is at $x=3$.

b $f(x)=x^2-6x+7$

c B is the y-intercept of the graph, and its coordinates are $\left(0,(-3)^2-2\right)=(0,7)$.

d By symmetry, $p=6$ as $6-3=3-0$.

6 **a** $h(x)=(x-2)^2-2(x-2)-3$

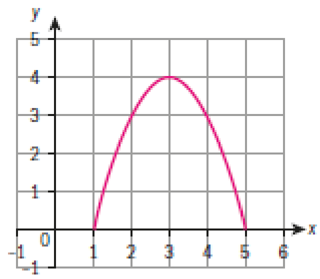
$=x^2-6x+5$

b The axis of symmetry lies at $x=-\frac{6}{2}=3$

c The vertex is at $(3,h(3))=(3,9-18+5)=(3,-4)$.

d $h(x)=(x-5)(x-1)$

e The graph is the same as that of $h(x)$, but reflected about the x-axis.



Exercise 3O

1 **a** The vertex is at $(2,-16)$ and the y-intercept is at $(0,-12)$. Thus $f(x)=a(x-2)^2-16$, and $-12=a(-2)^2-16=4a-16$. Thus $a=1$. In standard form,

$f(x)=(x-2)^2-16=x^2-4x+4-16$

$=x^2-4x-12$

b $f(x)=a(x-1)(x+3)$ from the x-intercepts. $3=a*(-1)*3=-3a$. Thus $a=-1$. In standard form, $f(x)=-(x-1)(x+3)=-x^2-2x+3$.

c $f(x)=a(x-5)(x-1)$ from the x-intercepts. $-12=a*(-1)*3=-3a$. Thus $a=4$. In standard form, $f(x)=4(x-5)(x-1)=4x^2-24x+20$.

d The vertex is at $(2,-6)$. Thus $f(x)=a(x-2)^2-6$, and $6=a(2)^2-6=4a-6$. Thus $a=3$. In standard form, $f(x)=3(x-2)^2-6=3x^2-12x+6$.

e $f(x)=a(x-2)(x+5)$ from the x-intercepts. $3=a*(-1)*6=-6a$. Thus $a=-\frac{1}{2}$. In standard form, $f(x)=-\frac{1}{2}(x-2)(x+5)=-\frac{1}{2}x^2-\frac{3}{2}x+5$

f The vertex is at $(-10,60)$. Thus $f(x)=a(x+10)^2+60$, and $45=a(-5)^2+60=25a+60$. Thus $a=-\frac{3}{5}$. In standard form, $f(x)=-\frac{3}{5}(x+10)^2+60=-\frac{3}{5}x^2-12x$.

2 a In intercept form, $f(x)=a(x-3)(x+1)$ Therefore, the axis of symmetry is at $x=\frac{3-1}{2}=1$.

b The vertex is at $(1,4)$ as $x=1$ is the axis of symmetry and 4 the maximum value.

c Since the vertex is at $(1,4)$, $h=1$ and $k=4$. So $f(x)=a(x-1)^2+4$. As we also know that $f(3)=0$, $4a+4=0$ and thus $a=-1$. So $f(x)=-(x-1)^2+4$

d $g(x)=f(x-4)-5$

$=-(x-5)^2-1$

$=-(x^2-10x+25)-1$

$=-x^2+10x-26$

3 a The vertex is at $(4,80)$. The model rocket is predicted to reach a maximum of 80 m, 4 s after it is launched.

b In intercept form, $h(t)=at(t-8)$. Inserting the coordinates of the vertex, we obtain $80=a\times4\times(-4)=-16a$.

Thus $a=-5$. Overall, $h(t)=-5t(t-8)$ $0\le t\le 8$

c $h(2.4)=-5\times2.4\times(-5.6)=67.2$. Therefore, the rocket is predicted to be 67.20 metres high.

Exercise 3P

1 a $x^2-4x+3=(x-3)(x-1)$. Thus $x=1$ or $x=3$.

b $x^2-x-20=(x-5)(x+4)$. Thus $x=5$ or $x=-4$.

c $x^2-8x+12=(x-6)(x-2)$. Thus $x=2$ or $x=6$.

d $x^2-121=(x-11)(x+11)$. Thus $x=11$ or $x=-11$.

e $x^2+x-42=(x-6)(x+7)$. Thus $x=6$ or $x=-7$.

f $x^2-8x+16=(x-4)^2$. Thus $x=4$.

2 a $2x^2+x-3=(2x+3)\ (x-1)$. Thus $x=1$ or $x=-\frac{3}{2}$.

b $3x^2+5x-12=(3x-4)\ (x+3)$. Thus $x=\frac{4}{3}$ or $x=-3$.

c $4x^2+11x+6=(x+2)\ (4x+3)$. Thus $x=-2$ or $x=-\frac{3}{4}$.

d $9x^2-49=\left(x-\frac{7}{3}\right)\ \left(x+\frac{7}{3}\right)$. Thus $x=\frac{7}{3}$ or $x=-\frac{7}{3}$.

e $4x^2-16x+7=(2x-7)(2x-1)$. Thus $x=\frac{7}{2}$ or $x=\frac{1}{2}$.

f $12x^2+11x-5=(3x-1)(4x+5)$. Thus $x=\frac{1}{3}$ or $x=-\frac{5}{4}$.

Exercise 3Q

1 **a** $\left(x^2 - x - 20\right) - (2x + 8) = x^2 - 3x - 28$

$= (x - 7)(x + 4)$

Thus $x = 7$ or $x = -4$.

b $\left(2x^2 - 3x - 8\right) - \left(-x^2 + 2x\right)$

$= 3x^2 - 5x - 8 = (3x - 8)(x + 1)$

Thus $x = \frac{8}{3}$ or $x = -1$.

c $\left(4x^2 + 20\right) - \left(3x^2 + 10x - 4\right)$

$= x^2 - 10x + 24 = (x - 6)(x - 4)$

Thus $x = 4$ or $x = 6$.

d $\left(3x^2 + 15x\right) + (x + 5)$

$= 3x^2 + 16x + 5 = (3x + 1)(x + 5)$

Thus $x = -\frac{1}{3}$ or $x = -5$.

e $3(x + 2)(x - 2) - (5x)$

$= 3x^2 - 5x - 12 = (3x + 4)(x - 3)$

Thus $x = -\frac{4}{3}$ or $x = 3$.

f For $x \neq 0$, $x + 8 = \frac{-15}{x}$ if and only if

$x^2 + 8x = -15$.

$x^2 + 8x + 15 = (x + 3)(x + 5)$ and thus

$x = -3$ or $x = -5$.

2 **a** $(f \circ g)(x) = (2x + 1)^2 - 2$

$= 4x^2 + 4x + 1 - 2 = 4x^2 + 4x - 1$

b $\left(4x^2 + 4x - 1\right) - \left(x^2 + 5x + 3\right)$

$= 3x^2 - x - 4 = (3x - 4)(x + 1)$

Thus $x = \frac{4}{3}$ or $x = -1$.

Exercise 3R

1 $x^2 - 8x + 16 = (x - 4)^2 = 10$. Thus

$x = \pm\sqrt{10} + 4$.

2 $x^2 + 20x + 100 = (x + 10)^2 = 15$. Thus

$x = \pm\sqrt{15} - 10$.

3 $x^2 + 12x + 36 = (x + 6)^2 = 12$. Thus

$x = \pm\sqrt{12} - 6$.

4 $x^2 - 10x + 25 = (x - 5)^2 = 27$. Thus

$x = \pm\sqrt{27} + 5$.

5 $4x^2 + 3x + 2 = -x + 5$

$4x^2 + 4x = 3$

$4\left(x^2 + x\right) = 3$

$4\left(x + \frac{1}{2}\right)^2 = 4$

$\left(x + \frac{1}{2}\right)^2 = 1$

$x = -\frac{1}{2} \pm 1$

$f\left(-\frac{3}{2}\right) = \frac{3}{2} + 5 = 6.5$

$f\left(\frac{1}{2}\right) = -\frac{1}{2} + 5 = 4.5$

6 (1.18, 7.35), (−1.96, 1.07)

7 (1, 5)

8 (2.72, 7.64), (0.613, −0.0872)

9 $x = -0.802, 1.80$

10 $x = -2.91, 0.915$

Exercise 3S

1 $\left(\frac{b}{2}\right)^2 = 6^2 = 36$. Therefore consider

$x^2 + 12x + 36 = 2 + 36 = 38$. This factorises

to $(x + 6)^2 = 38$, giving $x = -6 \pm \sqrt{38}$

2 $\left(\frac{b}{2}\right)^2 = \left(-\frac{3}{2}\right)^2 = \frac{9}{4}$. Therefore consider

$x^2 - 3x + \frac{9}{4} = 2 + \frac{9}{4} = \frac{17}{4}$. This factorises to

$\left(x - \frac{3}{2}\right)^2 = \frac{17}{4}$, giving

$x = \frac{\pm\sqrt{17}}{\sqrt{4}} + \frac{3}{2} = \frac{3 \pm \sqrt{17}}{2}$

3 $x^2 - 6x + 4 = 0$ is equivalent to

$x^2 - 6x = -4$. $\left(\frac{b}{2}\right)^2 = (-3)^2 = 9$. Therefore

consider $x^2 - 6x + 9 = -4 + 9 = 5$. This

factorises to $(x - 3)^2 = 5$, giving $x = 3 \pm \sqrt{5}$

4 $x^2 - 12x + 4 = 0$ is equivalent to

$x^2 - 12x = -4$. $\left(\frac{b}{2}\right)^2 = (-6)^2 = 36$.

Therefore consider

$x^2 - 12x + 36 = -4 + 36 = 32$. This

factorises to $(x-6)^2 = 32$, giving

$x = 6 \pm \sqrt{32} = 6 \pm 4\sqrt{2}$

5 $x^2 + 5x - 4 = 0$ is equivalent to $x^2 + 5x = 4$

$\left(\frac{b}{2}\right)^2 = \left(\frac{5}{2}\right)^2 = \frac{25}{4}$. Therefore consider

$x^2 + 5x + \frac{25}{4} = 4 + \frac{25}{4} = \frac{41}{4}$. This factorises

to $\left(x + \frac{5}{2}\right)^2 = \frac{41}{4}$, giving

$x = \frac{\pm\sqrt{41}}{\sqrt{4}} - \frac{5}{2} = \frac{-5 \pm \sqrt{41}}{2}$

6 $x^2 + x - 11 = 0$ is equivalent to $x^2 + x = 11$

$\left(\frac{b}{2}\right)^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$. Therefore consider

$x^2 + x + \frac{1}{4} = 11 + \frac{1}{4} = \frac{45}{4}$. This factorises to

$\left(x + \frac{1}{2}\right)^2 = \frac{45}{4}$, giving

$x = \frac{\pm\sqrt{45}}{\sqrt{4}} - \frac{1}{2} = \frac{-1 \pm \sqrt{45}}{2} = \frac{-1 \pm 3\sqrt{5}}{2}$

Exercise 3T

1 $2x^2 + 16x = 10$ is equivalent to

$x^2 + 8x = 5$. $\left(\frac{b}{2}\right)^2 = 4^2 = 16$. Therefore

consider $x^2 + 8x + 16 = 5 + 16 = 21$. This

factorises to $(x+4)^2 = 21$, giving

$x = -4 \pm \sqrt{21}$.

2 $5x^2 - 30x = 10$ is equivalent to

$x^2 - 6x = 2$. $\left(\frac{b}{2}\right)^2 = (-3)^2 = 9$. Therefore

consider $x^2 - 6x + 9 = 2 + 9 = 11$. This

factorises to $(x-3)^2 = 11$, giving

$x = 3 \pm \sqrt{11}$.

3 $6x^2 - 12x - 3 = 0$ is equivalent to

$x^2 - 2x = \frac{1}{2}$. $\left(\frac{b}{2}\right)^2 = (-1)^2 = 1$. Therefore

consider $x^2 - 2x + 1 = \frac{1}{2} + 1 = \frac{3}{2}$. This

factorises to $(x-1)^2 = \frac{3}{2}$, giving

$x = 1 \pm \sqrt{\frac{3}{2}}$.

4 $6x(x+8) = 12$ is equivalent to

$x(x+8) = x^2 + 8x = 2$. $\left(\frac{b}{2}\right)^2 = 4^2 = 16$.

Therefore consider

$x^2 + 8x + 16 = 2 + 16 = 18$. This factorises

to $(x+4)^2 = 18$, giving

$x = -4 \pm \sqrt{18} = -4 \pm 3\sqrt{2}$.

5 $2x^2 + x - 6 = 0$ is equivalent to $x^2 + \frac{1}{2}x = 3$

. $\left(\frac{b}{2}\right)^2 = \left(\frac{1}{4}\right)^2 = \frac{1}{16}$. Therefore consider

$x^2 + \frac{1}{2}x + \frac{1}{16} = 3 + \frac{1}{16} = \frac{49}{16}$. This factorises

to $\left(x + \frac{1}{4}\right)^2 = \frac{49}{16}$, giving

$x = \frac{\pm\sqrt{49}}{\sqrt{16}} - \frac{1}{4} = \frac{-1 \pm \sqrt{49}}{4} = \frac{-1 \pm 7}{4}$. This

means that x is either $\frac{3}{2}$ or -2.

6 $2x(x+8) + 12 = 0$ is equivalent to

$x(x+8) = x^2 + 8x = -6$. $\left(\frac{b}{2}\right)^2 = 4^2 = 16$.

Therefore consider

$x^2 + 8x + 16 = -6 + 16 = 10$. This factorises

to $(x+4)^2 = 10$, giving $x = -4 \pm \sqrt{10}$.

7 **a** Revenue is equal to cost when
$R(x) = C(x)$, i. e. when

$35x - 0.25x^2 = 300 + 15x$.

b This is equivalent to
$-0.25x^2 + 20x = 300$, which is in turn
equivalent to $x^2 - 80x = -1200$.

$\left(\frac{b}{2}\right)^2 = (-40)^2 = 1600$. Therefore

consider
$x^2 - 80x + 1600 = -1200 + 1600 = 400$.

This factorises to $(x-40)^2 = 400$,

giving $x = 40 \pm \sqrt{400} = 40 \pm 20 = 20, 60$.

c The break-even points lie at $x = 20$ and $x = 60$.

d We will want to find where the
maximum of the equation
$P(x) = R(x) - C(x)$ lies. This will just
be the coordinates of the vertex, since
the leading coefficient is negative.

$P(x) = R(x) - C(x)$

$= -0.25x^2 + 20x - 300$

In vertex form, this is $P(x) = -0.25(x-40)^2 + 100$. Thus, the maximal profit is reached at 40 subscribers.

e As seen from the vertex form above, the vertex has coordinates (40,100) and therefore the maximal profit is equal to 100 thousand Euros.

Exercise 3U

1 a $x = \frac{-4 \pm \sqrt{16+8}}{2} = -2 \pm \frac{\sqrt{24}}{2} = -2 \pm \sqrt{6}$

b $x = \frac{8 \pm \sqrt{64-60}}{6} = \frac{8 \pm 2}{6}$; that is, $x = 1$ or $x = \frac{5}{3}$.

c $x = \frac{5 \pm \sqrt{25+16}}{4} = \frac{5 \pm \sqrt{41}}{4}$

2 a $x^2 + 3x - 9 = 0$. Thus
$x = \frac{-3 \pm \sqrt{9+36}}{2} = -\frac{-3 \pm \sqrt{45}}{2}$
$= \frac{-3 \pm 3\sqrt{5}}{2}$

b $3x^2 - 4x - 2 = 0$. Thus
$x = \frac{4 \pm \sqrt{16+24}}{6} = -\frac{4 \pm \sqrt{40}}{6} = \frac{2 \pm \sqrt{10}}{3}$

c $-x^2 + 2x + 2 = 0$. Thus
$x = \frac{-2 \pm \sqrt{4+8}}{-2} = 1 \pm \sqrt{3}$

d $3x^2 + 4x + 10 = 0$. Thus
$x = \frac{-4 \pm \sqrt{16-120}}{6} = \frac{-4 \pm \sqrt{-104}}{6}$. As -104 has no real square root, the equation has no real solution.

e $-2x^2 + 10x - 9 = 0$. Thus
$x = \frac{-10 \pm \sqrt{100-72}}{-4} = \frac{5 \pm \sqrt{7}}{2}$

f $2x^2 - 9x + 9 = 0$. Thus
$x = \frac{9 \pm \sqrt{81-72}}{4} = -\frac{9 \pm 3}{4}$; that is,
$x = 3$ or $x = \frac{3}{2}$.

g $(x+3)(x+1) = 2x(x-1)$. This is equivalent to $x^2 + 4x + 3 = 2x^2 - 2x$, which simplifies to $x^2 - 6x - 3 = 0$. Thus
$x = \frac{6 \pm \sqrt{36+12}}{2} = 3 \pm \sqrt{12} = 3 \pm 2\sqrt{3}$.

3 a $x = \frac{-5 \pm \sqrt{25+144}}{12} = \frac{-5 \pm \sqrt{169}}{12}$
$= \frac{-5 \pm 13}{12}$; that is, $x = \frac{2}{3}$ or $x = -\frac{3}{2}$.

b $x = \frac{4 \pm \sqrt{16-8}}{4} = -\frac{4 \pm \sqrt{8}}{4} = \frac{2 \pm \sqrt{2}}{2}$

c $x = \frac{-2 \pm \sqrt{4+16}}{-2} = 1 \pm \sqrt{5}$

4 a $c = -2$

b $2x^2 - 4x - 2 = 2(x^2 - 2x - 1)$
$= 2(x-1)^2 - 4$. Therefore the vertex is at $(1, -4)$.

c Using the quadratic formula:
$x = \frac{4 \pm \sqrt{16+16}}{4} = 1 \pm \sqrt{2}$. Therefore $r = 1$ and $s = 2$.

Exercise 3V

1 a $\Delta = (-5)^2 - 4 \times 1 \times 9 = 25 - 36 = -11$.
Therefore the equation has no real roots.

b $\Delta = 7^2 - 4 \times 6 \times (-3) = 49 + 72 = 121$.
Therefore the equation has two distinct real roots.

c $\Delta = (-4)^2 - 4 \times 1 \times 15 = 16 - 60 = -44$.
Therefore the equation has no real roots.

d $\Delta = 4^2 - 4 \times 3 \times (-8) = 16 + 96 = 112$.
Therefore the equation has two distinct real roots.

e $\Delta = (-4)^2 - 4 \times 1 \times 4 = 16 - 16 = 0$.
Therefore the equation has two equal real roots.

f $\Delta = (-1)^2 - 4 \times 5 \times 10 = 1 - 200 = -199$.
Therefore the equation has no real roots.

2 a $\Delta = 3^2 - 4k = 9 - 4k$. This is positive whenever $k < \frac{9}{4}$.

b $\Delta = 20^2 - 20k = 400 - 20k$. This is positive whenever $k < 20$.

3 a $\Delta = 5^2 - 4p = 25 - 4p$. This is 0 if and only if $p = \frac{25}{4}$.

b $\Delta = (-12)^2 - 12p = 144 - 12p$. This is 0 if and only if $p = 12$.

c $\Delta = (-2p)^2 - 32 = 4p^2 - 32$. This is 0 if and only if $p^2 = 8$, which holds for $p = \pm\sqrt{8} = \pm 2\sqrt{2}$.

d $\Delta = (-3p)^2 + 8p = 9p^2 + 8p = p(9p + 8)$. This is 0 if and only if $p = 0$ or $p = -\frac{8}{9}$.

4 a $\Delta = (-2)^2 - 4m = 4 - 4m$. This is negative if and only if $m > 1$.

b $\Delta = (-6)^2 - 12m = 36 - 12m$. This is negative if and only if $m > 3$.

c $\Delta = 5^2 - 4(m - 2) = 33 - 4m$. This is negative if and only if $m > \frac{33}{4}$.

Exercise 3W

1 a We need to find the x-intercepts. By the quadratic formula,
$x = \frac{-5 \pm \sqrt{25 + 24}}{6} = \frac{-5 \pm 7}{6}$. Since the coefficient of x^2 is positive, the parabola will be concave up. Thus the inequality is satisfied whenever $x \le -2$ or $x \ge \frac{1}{3}$

b $x^2 \le 5$ if and only if $-\sqrt{5} \le x \le \sqrt{5}$.

c This is equivalent to $x^2 + 4x - 6 < 0$. By the quadratic formula,
$x = \frac{-4 \pm \sqrt{16 + 24}}{2} = -2 \pm \sqrt{10}$.
As the parabola is concave up, the inequality is satisfied whenever $-2 - \sqrt{10} < x < -2 + \sqrt{10}$.

2 a $x \le -0.245$ or $x \ge 12.2$

b $-\frac{2}{3} \le x \le 3$

c $-0.890 \le x \le 1.26$

3 a $\Delta = k^2 - 16$. This is positive whenever $k > 4$ or $k < -4$.

b $\Delta = 4k^2 - 12$. This is positive whenever $k > \sqrt{3}$ or $k < -\sqrt{3}$.

4 $\Delta = 36m^2 - 4m = m(36m - 4)$. The zeroes of this equation are at $m = 0$ and $m = \frac{1}{9}$.

As the parabola described by Δ is concave up, this is negative if and only if $0 < m < \frac{1}{9}$

5 $\Delta = 36k^2 - 4k(k + 2) = 32k^2 - 8k$
$= 8k(4k - 1)$. The zeroes of this equation are at $k = 0$ and $k = \frac{1}{4}$. As the parabola described by Δ is concave up, this is positive if and only if $k < 0$ or $k > \frac{1}{4}$.

6 a $\Delta = p^2 - 48$

b As the graph has no x-intercepts, $p^2 - 48 < 0$. This means that $-\sqrt{48} < p < \sqrt{48}$, which can be simplified as $-4\sqrt{3} < p < 4\sqrt{3}$.

c As $6^2 = 36 < 48 < 49 = 7^2$, $m = 6$.

d $3x^2 + 6x + 4 = 3\left(x^2 + 2x + \frac{4}{3}\right)$

$= 3\left((x + 1)^2 + \frac{1}{3}\right) = 3(x + 1)^2 + 1$

Thus $a = 3$, $h = -1$ and $k = 1$

Exercise 3X

1 $24 = \frac{1}{2}h(2h + 4)$

$48 = 2h^2 + 4h$
$2h^2 + 4h - 48 = 0$
$h^2 + 2h - 24 = 0$
$(h + 6)(h - 4) = 0$
$h = 4, -6$
h must be positive
So $h = 4$ m
$b = 2h + 4 = 12$ m

2 a $h(3) = 2 + 20(3) - 4.9(3^2) = 17.9$ m

b $2 + 20t - 4.9t^2 = 6$

$4.9t^2 - 20t + 4 = 0$

$t = \frac{20 \pm \sqrt{400 - 78.4}}{9.8}$

$t = 0.211$ seconds, 3.87 seconds

c Maximum height when: $t = -\frac{b}{2a} = \frac{20}{9.8}$

$h = 2 + 20\left(\frac{20}{9.8}\right) - 4.9\left(\frac{20}{9.8}\right)^2$

$h = 22.4$ metres

3 a Fare $= 5.50 - 0.05x$

b Number of riders $= 800 + 10x$

c Revenue $= (5.50 - 0.05x)(800 + 10x)$

$= 4400 - 40x + 55x - 0.5x^2$
$= 4400 + 15x - 0.5x^2$

d $4400 + 15x - 0.5x^2 = 4500$

$0 = 0.5x^2 - 15x + 100$
$x = 10, 20$

10 or 20 decreases

e $4400 + 15x - 0.5x^2 > 0$

Using GDC: $x < 110$

4 a $y = -(x-2)^2 + 4 = -x(x-4)$

or $y = -x^2 + 4x$

b If the center of the object is aligned with the center of the archway, it spans form $x = 0.5$ to $x = 3.5$. Evaluating the function at $x = 0.5$ and $x = 3.5$ gives 1.75. Since 1.6 < 1.75, the object will fit through the archway.

5 a $A(x) = x(155 - x) = 155x - x^2$

b Maximum area occurs at:

$x = \frac{-b}{2a} = \frac{155}{2} = 77.5$

$w = \frac{310 - 2(77.5)}{2} = 77.5$

Dimension: 77.5 metres by 77.5 metres

c No; The touchline would not be longer than the goal line and 77.5 metres is less that the minimum of 90 metres for the touchline.

d $90 \le x \le 120$ (If the goal line restrictions are also taken into consideration the answer is $90 \le x \le 110$.

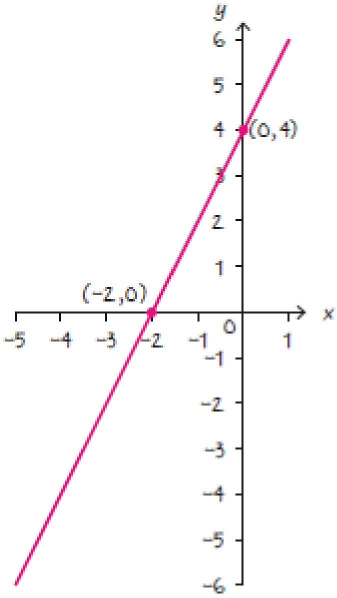
e Maximum occurs when $x = 90$

$w = \frac{310 - 2(90)}{2} = \frac{310 - 180}{2} = 65$

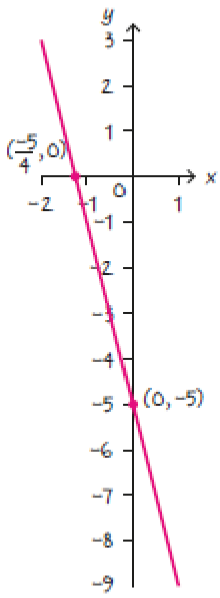
Area = $90 \times 65 =$ 5850 m^2

Chapter review

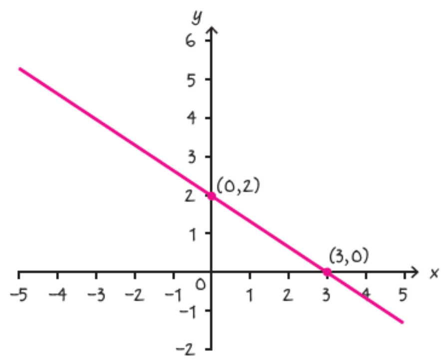
1 a



b



c



2 a $m = \frac{2 - -1}{-4 - 8} = \frac{3}{-12} = -\frac{1}{4}$

$y - 2 = -\frac{1}{4}(x + 4)$

$y - 2 = -\frac{1}{4}x - 1 \Rightarrow y = -\frac{1}{4}x + 1$

b $y = \frac{1}{2}x - 5$

c $m = \frac{-1}{\left(-\frac{2}{3}\right)} = \frac{3}{2}$

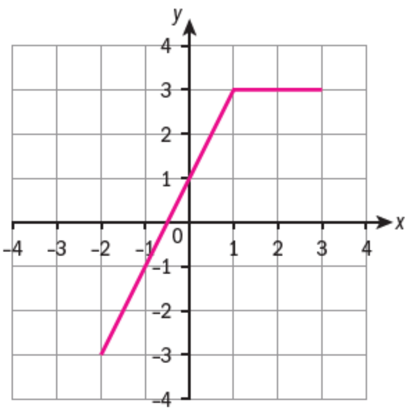
$y - 4 = \frac{3}{2}(x - 2)$

$y - 4 = \frac{3}{2}x - 3 \Rightarrow y = \frac{3}{2}x + 1$

d $y = -4$

3 a $f(1) = 3, f(2) = 3$

b



4 a Vertical stretch with scale factor 2, horizontal translation right 3

b Vertical dilation with scale factor $\frac{1}{2}$, vertical translation up 5

c Reflection in the x-axis, horizontal translation left 2, vertical translation down 1

d Horizontal dilation with scale factor $\frac{1}{3}$

e Reflection in the y-axis, vertical translation up 6

5 a $x-$intercepts: $2(x - 3)(x + 7) = 0$

$\Rightarrow x = 3, -7 \therefore (3,0), (-7,0)$

Axis of symmetry occurs at midpoint of x-intercepts

$x = \frac{3 + -7}{2} \Rightarrow x = -2$

b Found from the function

Axis of symmetry: $x = 4$, Vertex: $(4,2)$

c Axis of symmetry: $\frac{-b}{2a} = \frac{4}{-2} = -2$

$x = -2$

y-intercept found from the function: $(0,6)$

6 a $3x^2 + 18x + 20 = 3(x^2 + 6x) + 20$

$= 3((x + 3)^2 - 9) + 20$

$= 3(x + 3)^2 - 27 + 20$

$= 3(x + 3)^2 - 7$

i $a = 3$ **ii** $h = -3$ **iii** $k = -7$

b $(-3, -7)$

c $(-3 + 5, -7 - 3) = (2, -10)$

7 a $(x - 3)^2 = 64$

$x - 3 = \pm 8$

$x = -5, 11$

b $(x + 2)^2 = 7$

$x + 2 = \pm\sqrt{7}$

$x = -2 - \sqrt{7}, -2 + \sqrt{7}$

c $x^2 + 14x + 49 = 0$

$(x + 7)^2 = 0 \Rightarrow x = -7$

d $x^2 + x - 12 = 0$

$(x + 4)(x - 3) = 0 \Rightarrow x = -4, 3$

e $3x^2 + 4x - 7 = 0$

$(3x + 7)(x - 1) = 0 \Rightarrow x = 1, -\frac{7}{3}$

8 Equal real root: $b^2 - 4ac = 0$

$9k^2 - 16 = 0 \Rightarrow k^2 = \frac{16}{9} \Rightarrow k = -\frac{4}{3}, \frac{4}{3}$

9 From the x-intercepts:

$f(x) = a(x + 4)(x - 2) = ax^2 + 2ax - 8a$

From the y-intercept:

$-8a = -16 \Rightarrow a = 2$

$f(x) = 2x^2 + 4x - 16$

10 Using GDC solver

a $-0.679, 3.68$ **b** $-4.92, 1.42$

11 a $t = 0$, $h = 18$ m

b Maximum height occurs when:

$x = \frac{-b}{2a} = \frac{13}{9.8}$

$h = 18 + 13\left(\frac{13}{9.8}\right) - 4.9\left(\frac{13}{9.8}\right)^2$

$h = 26.6$ m

c $18 + 13t - 4.9t^2 = 0$

$t = -1.00, 3.66$ as $t > 0$

Time taken $= 3.66$ seconds

d $0 \le t \le 3.66$

e $18 + 13t - 4.9t^2 = 23$

$-4.9t^2 + 13t - 5 = 0$

$t = 0.4667.., 2.1863..$

$2.1863.. - 0.4667.. = 1.72$ seconds

12 a $A(-4,0)$; $B(0,7)$; $C(4,0)$

b $m = \dfrac{7-0}{0-4} = -\dfrac{7}{4}$

$y - 0 = -\dfrac{7}{4}(x-4) \Rightarrow y = -\dfrac{7}{4}x + 7$

c $2p$ cm by $-1.75p + 7$ cm

d Area $= 2p(-1.75p + 7) = -3.5p^2 + 14p$

e Maximum area occurs when

$p = \dfrac{-b}{2a} = \dfrac{14}{7} = 2$

So dimensions are 4 cm by 3.5 cm

f Area $= 4 \times 3.5 = 14$ cm^2

13 a $-7x - 12y + 168 = 0$

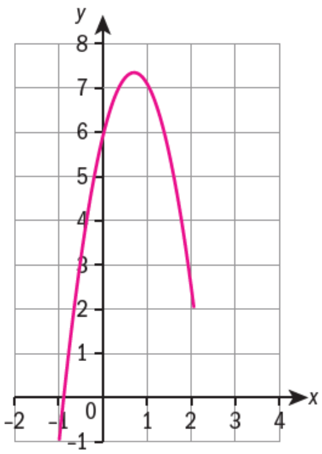
$12y = -7x + 168$ M1

$y = -\dfrac{7x}{12} + 14$ A1

b $A(24,0)$ and $B(0,14)$ A1A1

c Area $= \dfrac{1}{2} \times 24 \times 14 = 168$ units^2 M1A1

14 a



M1A1

b $(0, 5.9)$ and $(-0.885, 0)$ A1A1

c $-1.1 \le f(x) \le 7.35$ A1A1

15 a $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ M1

$x = \dfrac{6 \pm \sqrt{208}}{2}$ A1

$x = 3 \pm \sqrt{52}$ A1

$x = 3 \pm 2\sqrt{13}$ A1

b Using GDC

$3 - 2\sqrt{13} \le x \le 3 + 2\sqrt{13}$ M1A1

16 a $3(x-1)^2 - 18 = 3(x^2 - 2x + 1) - 18$

M1

$= 3x^2 - 6x - 15$ A1

b $(1, -18)$ A1

c $x = 1$ A1

d $f(x) \in \mathbb{R}$, $f(x) \ge -18$ A1A1

e $g(x) = 3((x-2)-1)^2 - 18 - 1$ M1A1

$= 3(x-3)^2 - 19$

$= 3(x^2 - 6x + 9) - 19$

$= 3x^2 - 18x + 8$ A1

17 a $8x^2 + 6x - 5 = 0$

$(4x+5)(2x-1) = 0$ M1A1

$4x + 5 = 0 \Rightarrow x = -\dfrac{5}{4}$ A1

$2x - 1 = 0 \Rightarrow x = \dfrac{1}{2}$ A1

b $8x^2 + 6x - 5 - k = 0$

No real solutions

$\Rightarrow b^2 - 4ac < 0$ M1

$36 - 4 \times 8 \times (-5 - k) < 0$ A1

$36 + 32(5 + k) < 0$

$5 + k < -\dfrac{36}{32} \Rightarrow k < -\dfrac{36}{32} - 5$

$k < -\dfrac{9}{8} - \dfrac{40}{8} \Rightarrow k < -\dfrac{49}{8}$ A1

18 a $x^2 - 10x + 27$

$= (x-5)^2 - 25 + 27$ M1A1

$= (x-5)^2 + 2$ A1

b Coordinates of the vertex is $(5,2)$

A1

c Equation of symmetry is $x = 5$ A1

19 a At $(10,0)$, $0=10^2+10b+c$, so

$10b+c=-100$ M1A1

Line of symmetry is $x=-\frac{b}{2}$, so $b=-5$

A1

Solving simultaneously gives

$-50+c=-100$

So $c=-50$ A1

Therefore the equation is

$y=x^2-5x-50$

b Setting $x=0$ gives the y-intercept of

$(0,-50)$ A1

Setting y = 0 and solving gives the x-intercept of $(-5,0)$ A1

20 a $f(x)=2\left[x^2-2x-4\right]$ M1

$=2\left[(x-1)^2-1-4\right]$ A1

$=2\left[(x-1)^2-5\right]$

$=2(x-1)^2-10$ A1

b A horizontal translation right 1 unit

A1

A vertical stretch with scale factor 2

A1

A vertical translation down 10 units

A1

21 a Two real roots $\Rightarrow b^2-4ac=0$ M1

$36-4(2k)(k)=0$ A1

$36-8k^2=0$

$k^2=\frac{36}{8}=\frac{9}{2}\Rightarrow k=\pm\frac{\sqrt{3}}{2}$ A1A1

b Equation of line of symmetry is

$x=-\frac{b}{2a}=-\frac{6}{4k}=-\frac{3}{2k}$ M1A1

Therefore $\frac{3}{2k}=1 \Rightarrow k=\frac{3}{2}$ A1

c $k=2\Rightarrow 4x^2+6x+2=0$

$2x^2+3x+1=0$

$(2x+1)(x+1)=0$ M1

$x=-\frac{1}{2}$ or $x=-1$ A1A1

22 a $A'(-6,10), B'(0,-16), C'(1,9)$

and $D'(7,-10)$ A4

b $A(12,13), B(0,-13), C(-2,12)$

and $D(-14,-7)$ A4

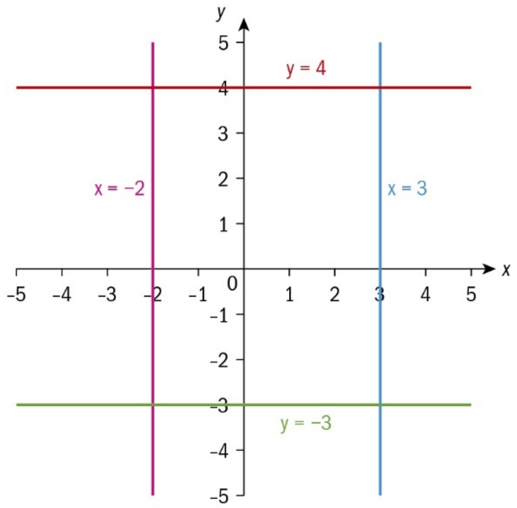
4 Equivalent representations: rational functions

Skills check

1 **a** $x = -5$ **b** $x = 6$

c $2x = 5 \Rightarrow x = \frac{5}{2}$

2



Exercise 4A

1 **a** $\frac{1}{3}$ **b** $\frac{1}{5}$ **c** $-\frac{1}{2}$

d $-\frac{1}{1} = -1$ **e** $\frac{5}{3}$ **f** $\frac{7}{22}$

g $-\frac{9}{8}$ **h** $\frac{1}{2\frac{3}{4}} = \frac{1}{\frac{2 \cdot 4 + 3}{4}} = \frac{4}{11}$

2 **a** $1.5 = \frac{3}{2} \Rightarrow \frac{1}{1.5} = \frac{2}{3}$ **b** $\frac{1}{x}$

c $\frac{1}{2x}$ **d** $\frac{1}{4y}$ **e** $\frac{4}{3x}$

f $\frac{t}{d}$ **g** $\frac{4d}{3}$ **h**

$\frac{x-3}{x+2}$

3 **a** $4 \cdot \frac{1}{4} = \frac{4}{4} = 1$

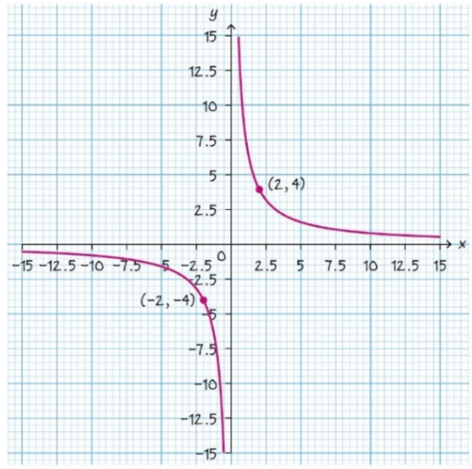
b $\frac{7}{11} \cdot \frac{11}{7} = \frac{7 \cdot 11}{7 \cdot 11} = \frac{77}{77} = 1$

c $\frac{2}{x} \cdot \frac{x}{2} = \frac{2x}{2x} = 1$

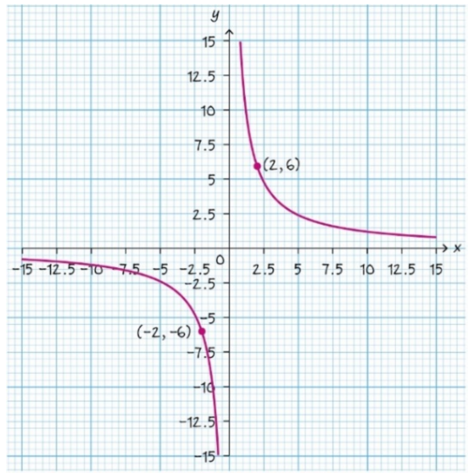
d $\frac{x-1}{x-2} \cdot \frac{x-2}{x-1} = \frac{(x-1)(x-2)}{(x-1)(x-2)} = 1$

Exercise 4B

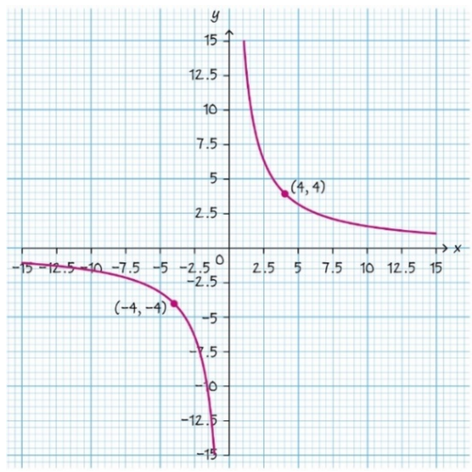
1 **a**



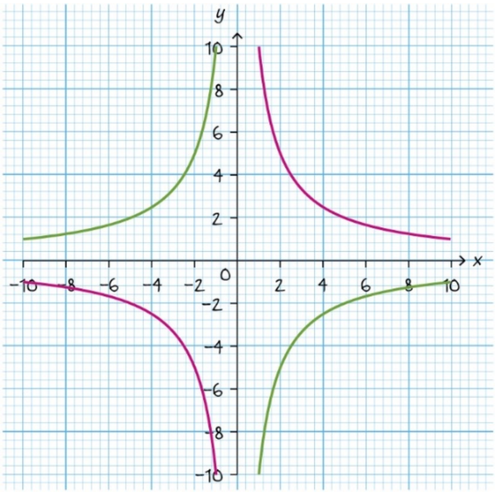
b



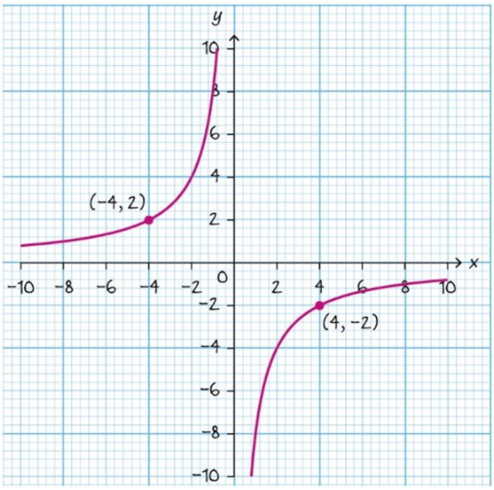
c



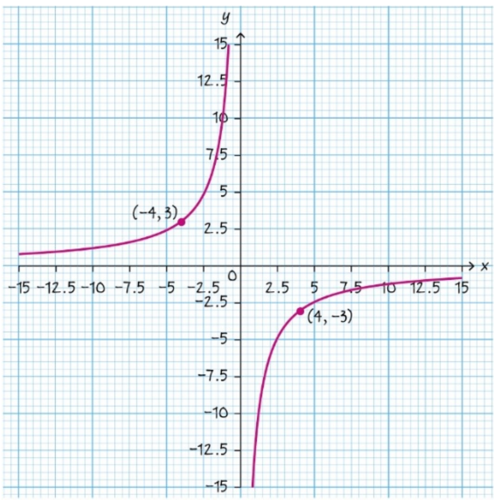
2



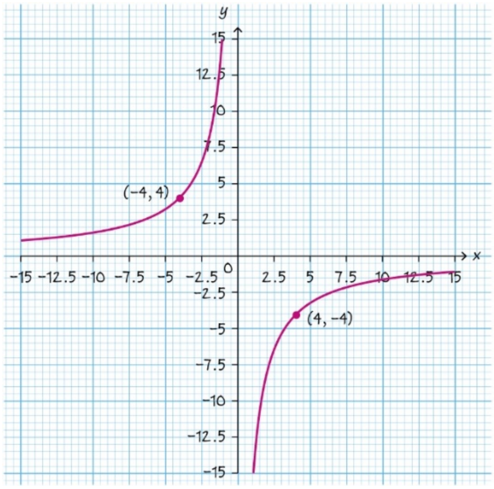
3 a



b



c

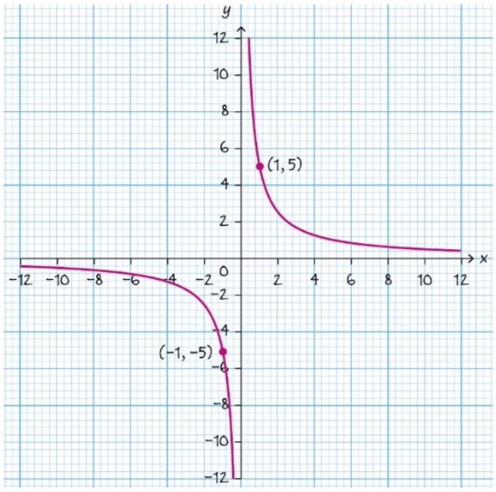


d The curves are in the opposite quadrants. The negative reflects the function in the x-axis.

4 x = 0, y = 0

Domain: $x \in$ ¡ $, x \neq 0$

Range: $y \in$ ¡ $, y \neq 0$



Exercise 4C

1 a $x = 2 \Rightarrow y = \frac{2}{x} = \frac{2}{2} = 1$

b $y = 4$

$y = \frac{2}{x}$

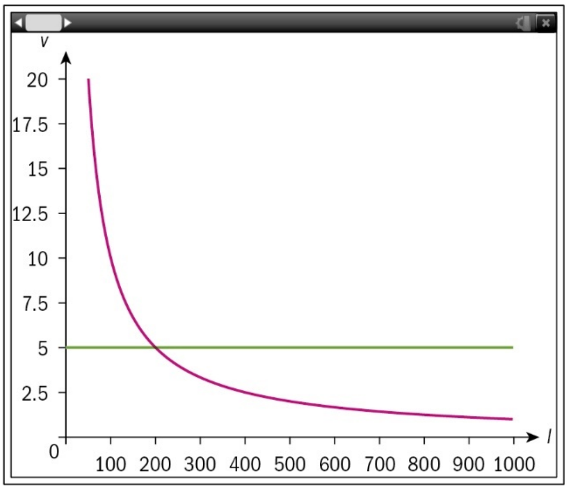
$\frac{2}{x} = 4$

$x = \frac{2}{4}$

$x = 0.5$

Chamse spends 30 seconds brushing her teeth.

2 a and **c**



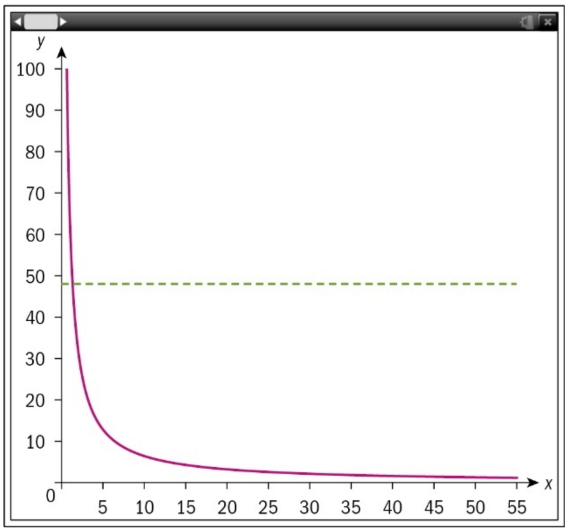
b $l = 10 \Rightarrow v = \dfrac{1000}{10} = 100$ Hz

c A string 5 cm long has vibrations of frequency 200 Hz.

3 a $y = \dfrac{64}{16} = 4$ videos of length 16 minutes

b $y = \dfrac{64}{x}$ is the equation that models the number of videos of x minutes.

c and **d**



1.33 minutes

Exercise 4D

1 a $y = \dfrac{1}{x+1}$

The vertical asymptote is at $x = -1$ and the horizontal asymptote at $y = 0$.

The domain is $x \in \mathbb{R}, x + 1 \neq 0 \Leftrightarrow x \neq -1$.

The range is $y \in \mathbb{R}, \mathbb{R} - \{0\}$.

b $y = \dfrac{1}{x-5}$

The vertical asymptote is at $x = -(-5) = 5$ and the horizontal asymptote at $y = 0$.

The domain is $x \in \mathbb{R}, x - 5 \neq 0 \Leftrightarrow x \neq 5$.

The range is $y \in \mathbb{R}, \mathbb{R} - \{0\}$.

c $y = \dfrac{-1}{x-4}$

The vertical asymptote is at $x - 4 = 0 \Leftrightarrow x = 4$ and the horizontal asymptote at $y = 0$.

The domain is $x \in \mathbb{R}, x - 4 \neq 0 \Leftrightarrow x \neq 4$.

The range is $y \in \mathbb{R}, \mathbb{R} - \{0\}$.

d $y = \dfrac{5}{x+5}$

The vertical asymptote is at $x + 5 = 0 \Leftrightarrow x = -5$ and the horizontal asymptote at $y = 0$.

The domain is $x \in \mathbb{R}, x + 5 \neq 0 \Leftrightarrow x \neq -5$.

The range is $y \in \mathbb{R}, \mathbb{R} - \{0\}$.

e $y = \dfrac{12}{x+1} + 2$

The vertical asymptote is at $x + 1 = 0 \Leftrightarrow x = -1$ and the horizontal asymptote at $y = 2$.

The domain is $x \in \mathbb{R}, x + 1 \neq 0 \Leftrightarrow x \neq -1$.

The range is $y \in \mathbb{R}, \mathbb{R} - \{2\}$.

f $y = \dfrac{12}{x+1} - 2$

The vertical asymptote is at $x + 1 = 0 \Leftrightarrow x = -1$ and the horizontal asymptote at $y = -2$.

The domain is $x \in \mathbb{R}, x + 1 \neq 0 \Leftrightarrow x \neq -1$.

The range is $y \in \mathbb{R}, \mathbb{R} - \{-2\}$.

g $y = \dfrac{4}{x-3} + 2$

The vertical asymptote is at $x - 3 = 0 \Leftrightarrow x = 3$ and the horizontal asymptote at $y = 2$.

The domain is $x \in \mathbb{R}, x - 3 \neq 0 \Leftrightarrow x \neq 3$.

The range is $y \in \mathbb{R}, \mathbb{R} - \{2\}$.

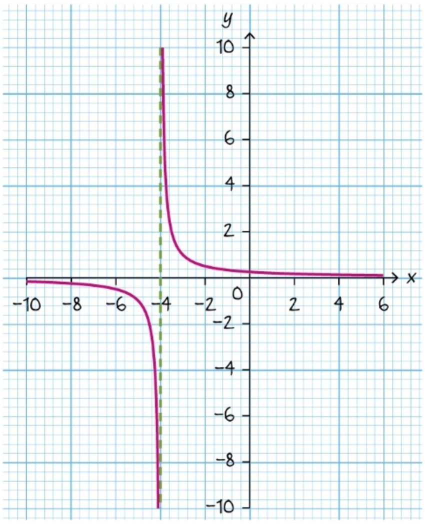
h $y = \dfrac{-4}{x-4} - 4$

The vertical asymptote is at $x - 4 = 0 \Leftrightarrow x = 4$ and the horizontal asymptote at $y = -4$.

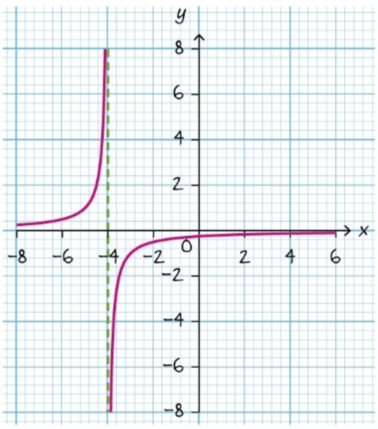
The domain is $x \in \mathbb{R}, x - 4 \neq 0 \Leftrightarrow x \neq 4$.

The range is $y \in \mathbb{R}, \mathbb{R} - \{-4\}$.

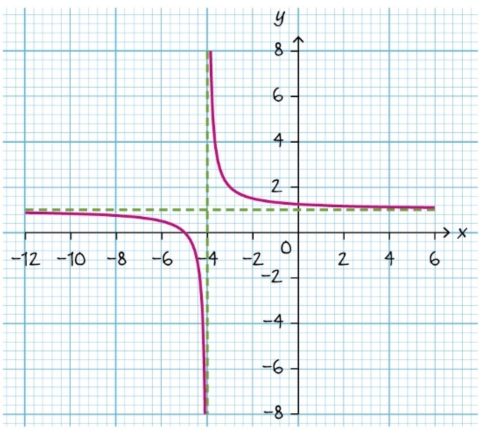
2 **a** $x \in \mathbb{R}, x \neq -4 \qquad y \in \mathbb{R}, y \neq 0$



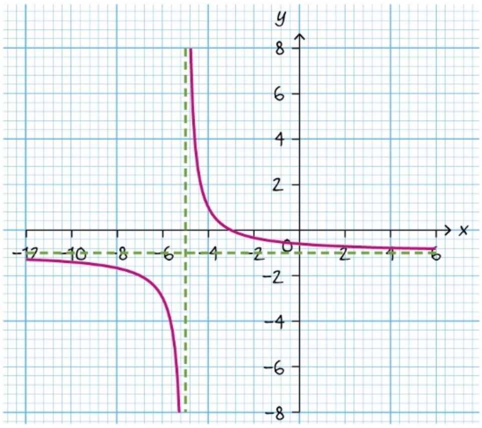
b $x \in \mathbb{R}, x \neq -4 \qquad y \in \mathbb{R}, y \neq 0$



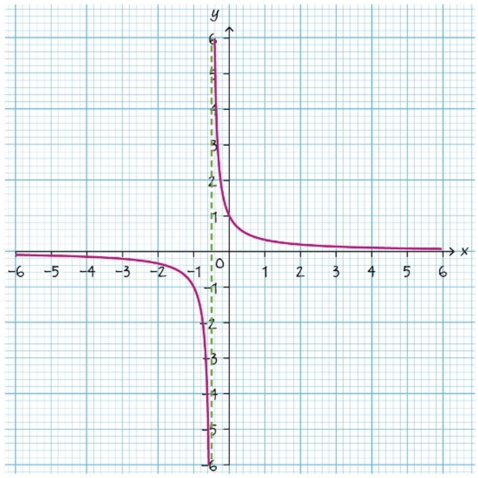
c $x \in \mathbb{R}, x \neq -4 \qquad y \in \mathbb{R}, y \neq 1$



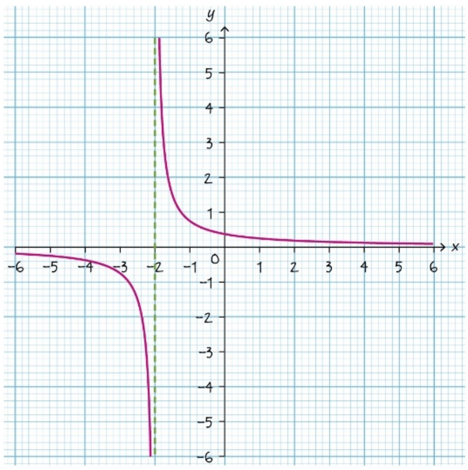
d $x \in \mathbb{R}, x \neq -5 \qquad y \in \mathbb{R}, y \neq 1$



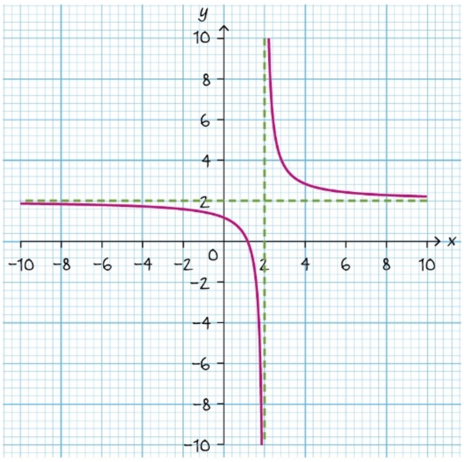
e $x \in \mathbb{R}, x \neq -0.5 \quad y \in \mathbb{R}, y \neq 0$



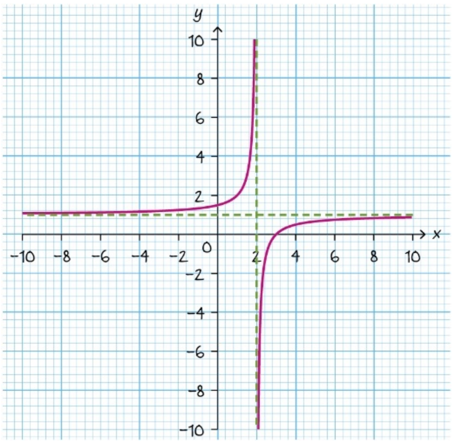
f $x \in \mathbb{R}, x \neq -2 \qquad y \in \mathbb{R}, y \neq 0$



g $x \in \mathbb{R}, x \neq 2$ $y \in \mathbb{R}, y \neq 2$



h $x \in \mathbb{R}, x \neq 2$ $y \in \mathbb{R}, y \neq 1$



3 a **2:** Translation of 2 units right

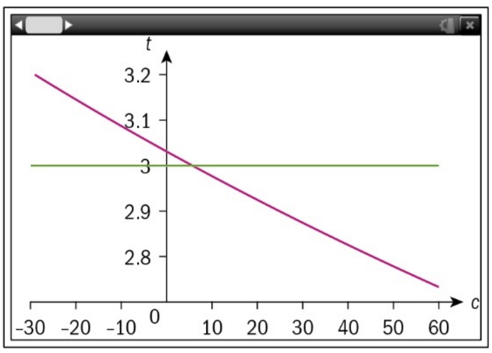
b **5:** Reflection in $y = 0$ and a translation of 2 units right

c **1:** Translation of 2 units right and 2 units up

d **4:** Translation of 2 units right and 2 units down

e **3:** Translation of 2 units right and vertical stretch by a factor of 3

4 a



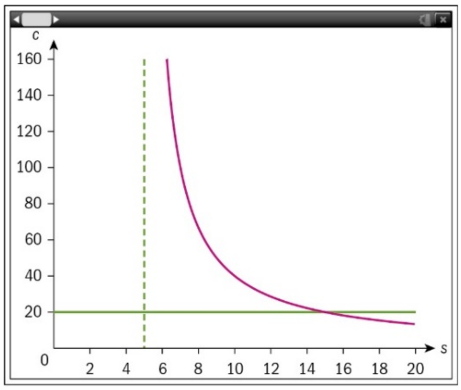
b 5.56

c $t = 6$

$$6 = \frac{1000}{0.6c + 330}$$
$$6(0.6c + 330) = 1000$$
$$3.6c + 1980 = 1000$$
$$c = \frac{1000 - 1980}{3.6}$$
$$c = -272.22°$$

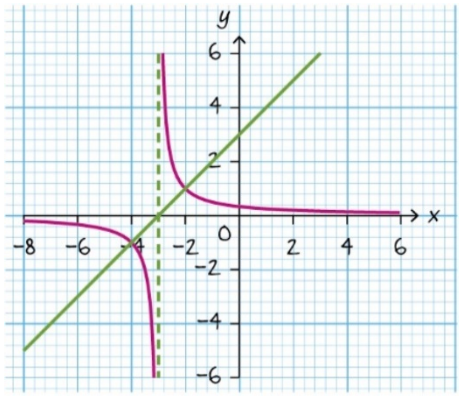
5 a $c = \dfrac{200}{s-5}$

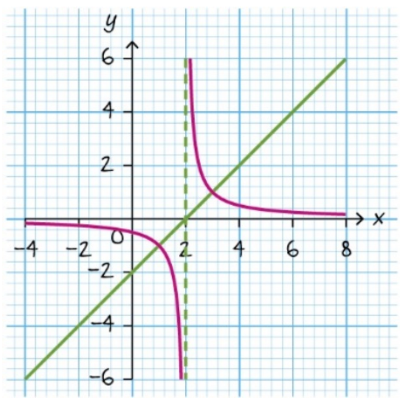
The vertical asymptote is at $s - 5 = 0 \Leftrightarrow s = 5$ and the horizontal asymptote at $c = 0$.



b 15 sessions.

6





The linear function is a line of symmetry for the rational function. The linear function crosses the x-axis at the same place as the vertical asymptote.

Exercise 4E

1 a $y = \dfrac{x+1}{x-1} \Rightarrow a = 1, b = 1, c = 1, d = -1$

The vertical asymptote is at $x = -\dfrac{d}{c} = -\dfrac{(-1)}{1} = 1$ and the horizontal asymptote at $y = \dfrac{a}{c} = \dfrac{1}{1} = 1$

Domain $x \in \mathbb{R}, x \neq 1$.

Range $y \in \mathbb{R}, y \neq 1$.

b $y = \dfrac{2x+3}{x+1} \Rightarrow a = 2, b = 3, c = 1, d = 1$

The vertical asymptote is at $x = -\dfrac{d}{c} = -\dfrac{1}{1} = -1$ and the horizontal asymptote at $y = \dfrac{a}{c} = \dfrac{2}{1} = 2$.

Domain $x \in \mathbb{R}, x \neq -1$.

Range $y \in \mathbb{R}, y \neq 2$.

c $y = \dfrac{6x-1}{2x+4} \Rightarrow a = 6, b = -1, c = 2, d = 4$

The vertical asymptote is at $x = -\dfrac{d}{c} = -\dfrac{4}{2} = -2$ and the horizontal asymptote at $y = \dfrac{a}{c} = \dfrac{6}{2} = 3$.

Domain $x \in \mathbb{R}, x \neq -2$.

Range $y \in \mathbb{R}, y \neq 3$.

d $y = \dfrac{2-3x}{5-4x} \Rightarrow a = -3, b = 2, c = -4, d = 5$

The vertical asymptote is at $x = -\dfrac{d}{c} = -\dfrac{5}{(-4)} = 1.25$ and the horizontal asymptote at $y = \dfrac{a}{c} = \dfrac{-3}{-4} = 0.75$.

Domain $x \in \mathbb{R}, x \neq 1.25$.

Range $y \in \mathbb{R}, y \neq 0.75$.

e $y = \dfrac{9x-2}{6-3x} \Rightarrow a = 9, b = -2, c = -3, d = 6$

The vertical asymptote is at $x = -\dfrac{d}{c} = -\dfrac{6}{(-3)} = 2$ and the horizontal asymptote at $y = \dfrac{a}{c} = \dfrac{9}{(-3)} = -3$.

Domain $x \in \mathbb{R}, x \neq 2$.

Range $y \in \mathbb{R}, y \neq -3$.

2 i B

$a = 1, b = -3, c = 1, d = 2$

Vertical asymptote: $x = -\dfrac{d}{c} = -2$

Horizontal asymptote: $y = \dfrac{a}{c} = 1$

ii A

$a = 0, b = 4, c = 1, d = 0$

Vertical asymptote: $x = -\dfrac{d}{c} = 0$

Horizontal asymptote: $y = \dfrac{a}{c} = 0$

iii D

$a = -2, b = 3, c = 1, d = 2$

Vertical asymptote: $x = -\dfrac{d}{c} = -2$

Horizontal asymptote: $y = \dfrac{a}{c} = -2$

iv C

$a = 2, b = -3, c = 1, d = 2$

Vertical asymptote: $x = -\dfrac{d}{c} = -2$

Horizontal asymptote: $y = \dfrac{a}{c} = 2$

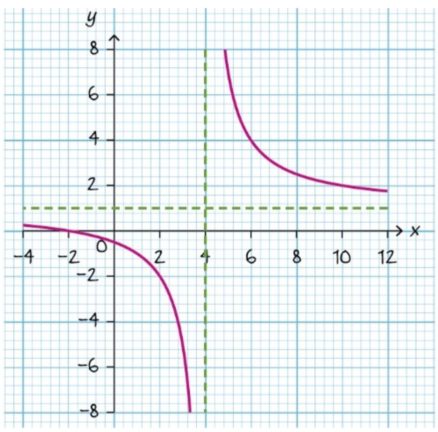
3 $y = \dfrac{x-p}{x-q} \Rightarrow a = 1, b = -p, c = 1, d = -q$

The vertical asymptote is at $x = -\dfrac{d}{c} = -\dfrac{(-q)}{1} = q$ and the horizontal asymptote at $y = \dfrac{a}{c} = \dfrac{1}{1} = 1$.

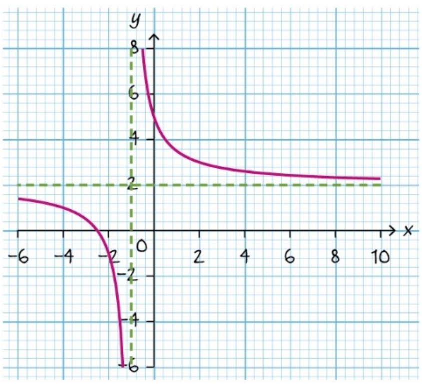
Domain $x \in \mathbb{R}, x \neq q$.

Range $y \in \mathbb{R}, y \neq 1$.

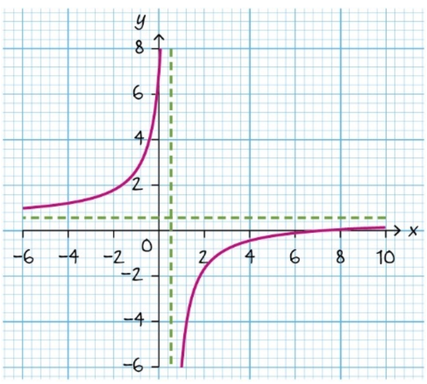
4 a



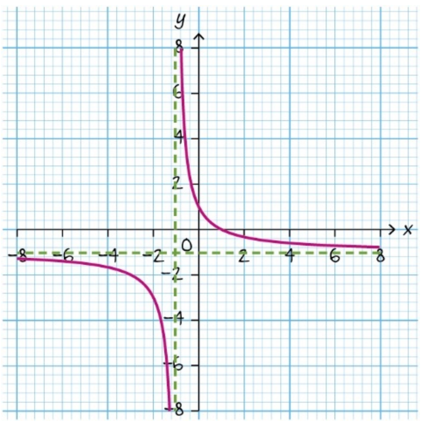
b



c



d



5 **a** $\frac{5}{2x}+\frac{x+7}{x+4}=2$

$\frac{5(x+4)+2x(x+7)}{2x(x+4)}=2$

$5x+20+2x^2+14x=4x(x+4)$

$2x^2+19x+20=4x^2+16x$

$2x^2-3x-20=0$

$2x^2-8x+5x-20=0$

$2x(x-4)+5(x-4)=0$

$(x-4)(2x+5)=0$

So $x=4$ and $x=\frac{-5}{2}$.

b $\frac{2x-3}{x+1}=\frac{x+6}{x-2}$

$(2x-3)(x-2)=(x+1)(x+6)$

$2x^2-3x-4x+6=x^2+6x+x+6$

$x^2-14x=0$

$x(x-14)=0$

So $x=0$ and $x=14$.

c $7-\frac{5}{x-2}=\frac{10}{x+2}$

$\frac{7(x-2)-5}{x-2}=\frac{10}{x+2}$

$\frac{7x-19}{x-2}=\frac{10}{x+2}$

$(7x-19)(x+2)=10(x-2)$

$7x^2+14x-19x-38=10x-20$

$7x^2-15x-18=0$

$(x-3)(7x+6)=0$

So $x=3$ and $x=-\frac{6}{7}$.

d $\frac{x+5}{x+8}=1+\frac{6}{x+1}$

$\frac{x+5}{x+8}==\frac{x+1+6}{x+1}$

$\frac{x+5}{x+8}=\frac{x+7}{x+1}$

$(x+5)(x+1)=(x+8)(x+7)$

$x^2+6x+5=x^2+15x+56$

$9x+51=0$

$x=-\frac{51}{9}=-\frac{17}{3}$

6 $x=3$ is the extraneous solution. Therefore the solution to Will's equation is $x=2$.

7 **a** $f(x)=\frac{x+3}{x-2}$

$x=\frac{y+3}{y-2}$

$x(y-2)=y+3$

$xy-2x=y+3$

$xy-y=2x+3$

$y(x-1)=2x+3$

$y=\frac{2x+3}{x-1}$

$f^{-1}(x)=\frac{2x+3}{x-1}$

b $f(x)=\dfrac{7-2x}{x}$

$x=\dfrac{7-2y}{y}$

$xy=7-2y$

$y(x+2)=7$

$y=\dfrac{7}{x+2}$

$f^{-1}(x)=\dfrac{7}{x+2}$

c $f(x)=\dfrac{1+7x}{9-x}$

$x=\dfrac{1+7y}{9-y}$

$x(9-y)=1+7y$

$9x-xy=1+7y$

$y(7+x)=9x-1$

$y=\dfrac{9x-1}{7+x}$

$f^{-1}(x)=\dfrac{9x-1}{x+7}$

d $f(x)=\dfrac{5-11x}{x+6}$

$x=\dfrac{5-11y}{y+6}$

$x(y+6)=5-11y$

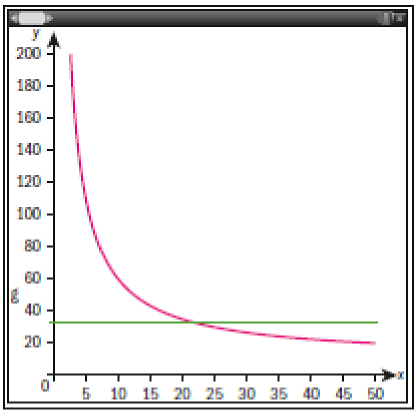
$xy+6x=5-11y$

$y(x+11)=5-6x$

$y=\dfrac{5-6x}{x+11}$

$f^{-1}(x)=\dfrac{5-6x}{x+11}$

8 **a** and **c**



b 20

c $M(s)=\dfrac{10s+500}{s}=20$

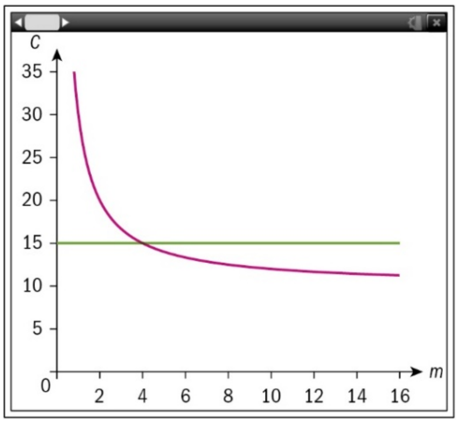
$10s+500=20s$

$500=10s$

$s=50$

9 **a** $C(m)=\dfrac{20+10m}{m}$ as 20 is the initial cost and them for every month there is another 10 AUD cost.

b



c 4 months

d The price will get closer to the horizontal asymptote $y=10$.

10 a $f(x)=m+\dfrac{6}{x-n}$

$=\dfrac{m(x-n)+6}{x-n}$

$=\dfrac{mx-mn+6}{x-n}$

$a=m$

$b=6-mn$

$c=1$

$d=-n$

The vertical asymptote is at

$x=-\dfrac{d}{c}=-\dfrac{(-n)}{1}=n=5.$

Hence $n=5$.

b $f(7)=7$

$f(7)=m+\dfrac{6}{7-5}=m+\dfrac{6}{2}$

$f(7)=m+3=7$

$m=4$

c The vertical asymptote is at

$x=\dfrac{a}{c}=\dfrac{4}{1}=4.$

11 a $y=\dfrac{4}{x-2}+3=\dfrac{4+3(x-2)}{x-2}=\dfrac{3x-2}{x-2}$

$a=3$

$b=-2$

$c=1$

$d=-2$

The horizontal asymptote is at

$y = \frac{a}{c} = \frac{3}{1} = 3.$

b The vertical asymptote is at

$x = -\frac{d}{c} = -\frac{(-2)}{1} = 2.$

c The x-intercept is when $y = 0$.

$\frac{3x-2}{x-2} = 0$

$3x = 2$ The point is

$x = \frac{2}{3}$

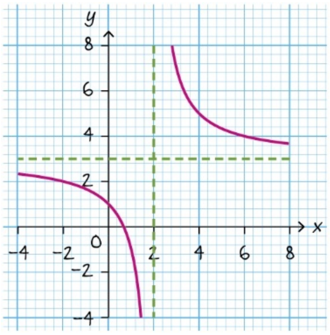
$(\frac{2}{3}, 0) = (0.667, 0).$

The y-intercept is when $x = 0$.

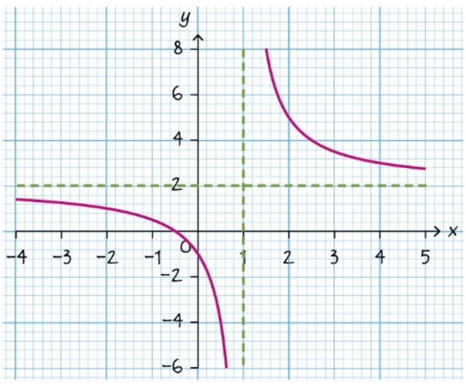
$\frac{3 \cdot 0 - 2}{0-2} = \frac{-2}{-2} = 1 = y$

The point is $(0,1)$.

d



12 a



b

$f(x) = \frac{2x+1}{x-1} \Rightarrow a = 2, b = 1, c = 1, d = -1$

The horizontal asymptote is at

$y = \frac{a}{c} = \frac{2}{1} = 2.$

The vertical asymptote is at

$x = -\frac{d}{c} = -\frac{(-1)}{1} = 1.$

c $f(x) = 0$

$\frac{2x+1}{x-1} = 0$

$2x + 1 = 0$

$x = -\frac{1}{2}$

The x-intercept of f is at point

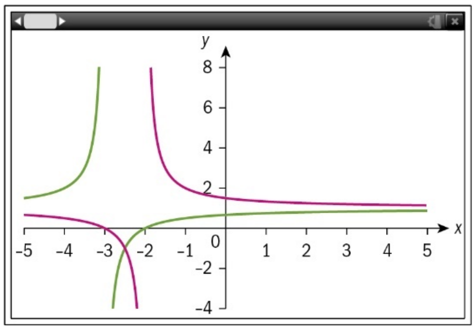
$(-\frac{1}{2}, 0) = (-0.5, 0).$

13 a $g \circ f(x) = g(f(x))$

$= g(\frac{x+2}{x+3}) = \frac{1}{\frac{x+2}{x+3}}$

$= \frac{x+3}{x+2}$

b



$x = -2.5$

Chapter review

1 a $y = \frac{2}{x} \Rightarrow a = 0, b = 2, c = 1, d = 0$

The horizontal asymptote is at

$y = \frac{a}{c} = \frac{0}{1} = 0.$

The vertical asymptote is at

$x = -\frac{d}{c} = -\frac{0}{1} = 0.$

Domain: $x \in \mathbb{R}, x \neq 0$

Range: $y \in \mathbb{R}, y \neq 0$

b $y = \frac{1}{x+8} \Rightarrow a = 0, b = 1, c = 1, d = 8$

The horizontal asymptote is at

$y = \frac{a}{c} = \frac{0}{1} = 0.$

The vertical asymptote is at

$x = -\frac{d}{c} = -\frac{8}{1} = -8.$

Domain: $x \in \mathbb{R}, x \neq -8$

Range: $y \in \mathbb{R}, y \neq 0$

c

$y = \dfrac{x}{2x - 10} \Rightarrow a = 1, b = 0, c = 2, d = -10$

The horizontal asymptote is at

$y = \dfrac{a}{c} = \dfrac{1}{2}.$

The vertical asymptote is at

$x = -\dfrac{d}{c} = -\dfrac{-10}{2} = 5.$

Domain: $x \in$ ¡ $, x \neq 5$

Range: $y \in$ ¡ $, y \neq \dfrac{1}{2}$

d $y = \dfrac{3}{x-2} + 3 = \dfrac{3 + 3(x-2)}{x-2} = \dfrac{3x-3}{x-2}$

$\Rightarrow a = 3, b = -3, c = 1, d = -2$

The horizontal asymptote is at

$y = \dfrac{a}{c} = \dfrac{3}{1} = 3.$

The vertical asymptote is at

$x = -\dfrac{d}{c} = -\dfrac{-2}{1} = 2.$

Domain: $x \in$ ¡ $, x \neq 2$

Range: $y \in$ ¡ $, y \neq 3$

e $y = \dfrac{2x}{x-9} \Rightarrow a = 2, b = 0, c = 1, d = -9$

The horizontal asymptote is at

$y = \dfrac{a}{c} = \dfrac{2}{1} = 2.$

The vertical asymptote is at

$x = -\dfrac{d}{c} = -\dfrac{-9}{1} = 9.$

Domain: $x \in$ ¡ $, x \neq 9$

Range: $y \in$ ¡ $, y \neq 2$

f $y = \dfrac{8x-5}{2x+4} \Rightarrow a = 8, b = -5, c = 2, d = 4$

The horizontal asymptote is at

$y = \dfrac{a}{c} = \dfrac{8}{2} = 4.$

The vertical asymptote is at

$x = -\dfrac{d}{c} = -\dfrac{4}{2} = -2.$

Domain: $x \in$ ¡ $, x \neq -2$

Range: $y \in$ ¡ $, y \neq 4$

g $y = \dfrac{1-x}{x+4} \Rightarrow a = -1, b = 1, c = 1, d = 4$

The horizontal asymptote is at

$y = \dfrac{a}{c} = \dfrac{-1}{1} = -1.$

The vertical asymptote is at

$x = -\dfrac{d}{c} = -\dfrac{4}{1} = -4.$

Domain: $x \in$ ¡ $, x \neq -4$

Range: $y \in$ ¡ $, y \neq -1$

h $y = \dfrac{2x-1}{2x+6} - 4 = \dfrac{2x - 1 - 4(2x+6)}{2x+6}$

$= \dfrac{2x - 1 - 8x - 24}{2x+6} = \dfrac{-6x-25}{2x+6}$

$\Rightarrow a = -6, b = -25, c = 2, d = 6$

The horizontal asymptote is at

$y = \dfrac{a}{c} = \dfrac{-6}{2} = -3.$

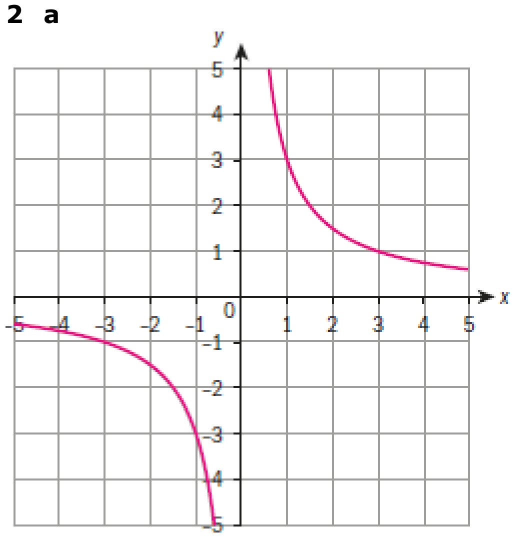
The vertical asymptote is at

$x = -\dfrac{d}{c} = -\dfrac{6}{2} = -3.$

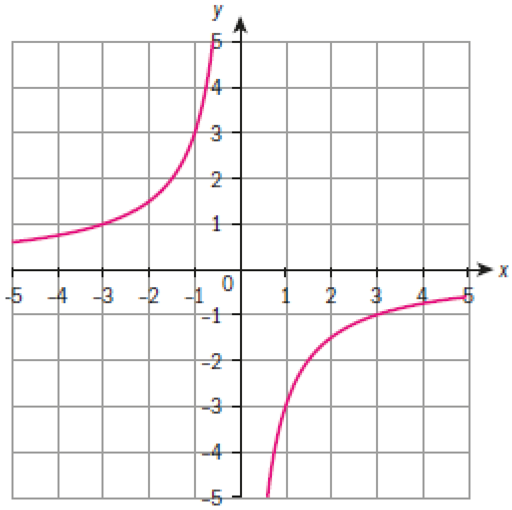
Domain: $x \in$ ¡ $, x \neq -3$

Range: $y \in$ ¡ $, y \neq -3$

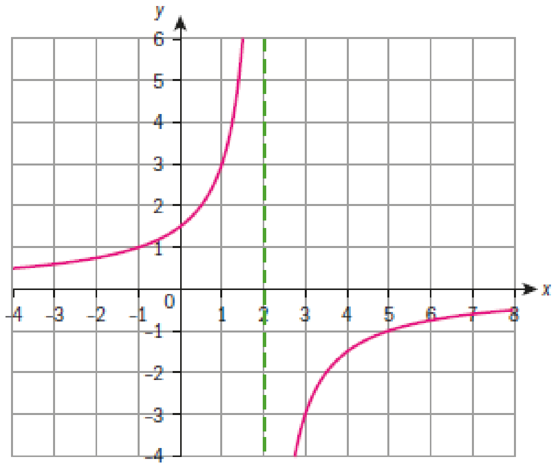
2 a



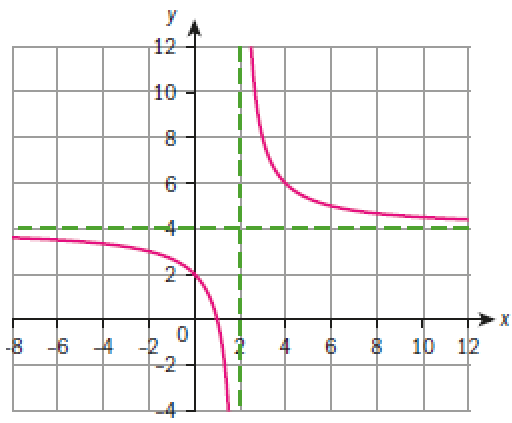
b



c



d



3 **a** $f(x)=\dfrac{1}{x-1}+2=\dfrac{1+2(x-1)}{x-1}=\dfrac{2x-1}{x-1}$

$\Rightarrow a=2, b=-1, c=1, d=-1$

The horizontal asymptote is at

$y=\dfrac{a}{c}=\dfrac{2}{1}=2.$

The vertical asymptote is at

$x=-\dfrac{d}{c}=-\dfrac{-1}{1}=1.$

b The x-intercept is $(\frac{1}{2},0)=(0.5,0)$ as:

$f(x)=0$

$\dfrac{2x-1}{x-1}=0$

$2x-1=0$

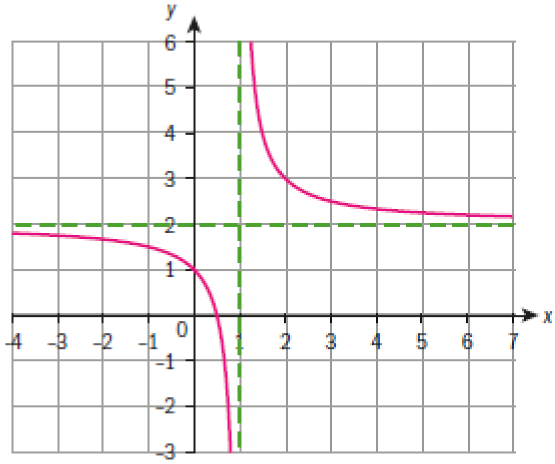
$x=\dfrac{1}{2}$

The y-intercept is $(0,1)$ as:

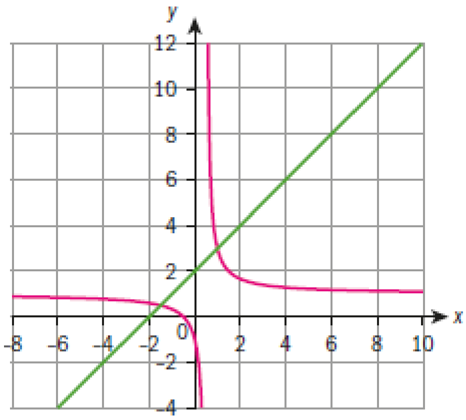
$x=0$

$f(0)=\dfrac{2\cdot 0-1}{0-1}=1$

c



4



x = −1.5, 1

5 **a** 1.29, 2.71 **b** 2.71 **c** 1.27

6 **a** $f(x)=0$

$\dfrac{2x-8}{1-x}=0$

$2x-8=0$

$x=\dfrac{8}{2}=4$

The x-intercept is therefore $(4, 0)$.

b $f(x) = \dfrac{2x-8}{1-x}$

$\Rightarrow a = 2, b = -8, c = -1, d = 1$

The vertical asymptote is at

$x = -\dfrac{d}{c} = -\dfrac{1}{-1} = 1.$

c The horizontal asymptote is at

$y = \dfrac{a}{c} = \dfrac{2}{-1} = -2.$

7 a $f(x) = \dfrac{ax+b}{x-d}$

The vertical asymptote is at

$x = -\dfrac{-d}{1} = d.$

The horizontal asymptote is at

$y = \dfrac{a}{1} = a.$

Hence $3 = d$ and $2 = a$.

b $f(1) = \dfrac{a+b}{1-d} = \dfrac{2+b}{1-3} = -4$

$f(1) = \dfrac{2+b}{-2} = -4$

$2 + b = 8$

$b = 6.$

8 a $f(x) = \dfrac{5}{x-m} + n = \dfrac{5+n(x-m)}{x-m}$

$= \dfrac{nx - mn + 5}{x-m}$

$a = n, b = -mn + 5, c = 1, d = -m$

$4 = -\dfrac{d}{c} = -\dfrac{-m}{1} = m$

b $f(0) = 7$

$f(0) = \dfrac{n \cdot 0 - 4n + 5}{0 - 4} = \dfrac{-4n+5}{-4} = 7$

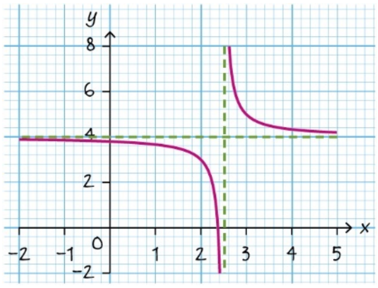
$4n - 5 = 28$

$4n = 33$

$n = \dfrac{33}{4}$

c $y = \dfrac{\frac{33}{4}}{1} = \dfrac{33}{4}$

9 a The x-intercept is $\left(-\dfrac{1}{2}, 0\right)$



b $x = 2.5$, $y = 4$

c 2.375 **d** 3.8

10 a $x = \dfrac{2y+1}{y-1}$

$x(y-1) = 2y + 1$

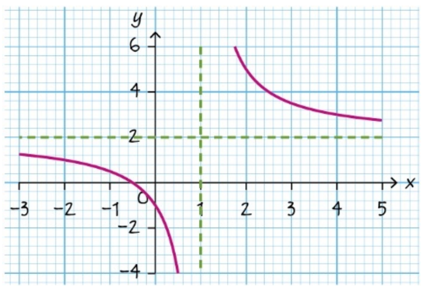
$xy - x = 2y + 1$

$y(x-2) = x + 1$

$y = \dfrac{x+1}{x-2}$

$f^{-1}(x) = \dfrac{x+1}{x-2}$

b



c $a = 2, b = 1, c = 1, d = -1$

The vertical asymptote is at

$x = -\dfrac{d}{c} = -\dfrac{-1}{1} = 1.$

The horizontal asymptote is at

$y = \dfrac{a}{c} = \dfrac{2}{1} = 2.$

d $f(x) = 0$

$\dfrac{2x+1}{x-1} = 0$

$2x + 1 = 0$

$x = -\dfrac{1}{2}$

The x-intercept is $(-\dfrac{1}{2}, 0)$.

e $f(x) = f^{-1}(x)$

$$\frac{2x+1}{x-1} = \frac{x+1}{x-2}$$

$$(2x+1)(x-2) = (x-1)(x+1)$$

$$2x^2 - 3x - 2 = x^2 - 1$$

$$x^2 - 3x - 1 = 0$$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{3 \pm \sqrt{9+4}}{2}$$

$$= \frac{3 \pm \sqrt{13}}{2} = \text{-}0.303, 3.30$$

11 a $f(x) = \dfrac{1}{x-2}$

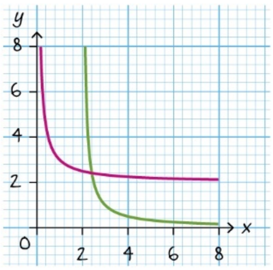
$$x = \frac{1}{y-2}$$

$$xy - 2x = 1$$

$$y = \frac{1+2x}{x}$$

$$f^{-1}(x) = \frac{1+2x}{x} = \frac{1}{x} + 2$$

b



c $\dfrac{1}{x-2} = \dfrac{1+2x}{x}$

$$x = (1+2x)(x-2)$$

$$x = x + 2x^2 - 2 - 4x$$

$$2x^2 - 4x - 2 = 0$$

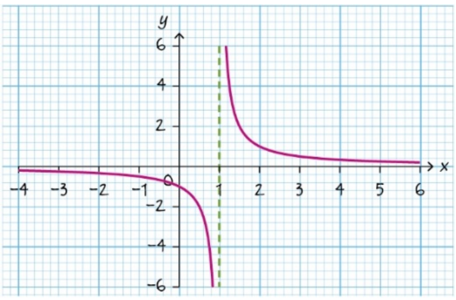
$$x^2 - 2x - 1 = 0$$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{2 \pm \sqrt{8}}{2}$$

$$x > 2$$

Hence the solution is $x = 2.41$.

12 a



b $g(x) = \dfrac{1}{x-3} + 3$

c $g(x) = 0$

$$\frac{1}{x-3} + 3 = 0$$

$$\frac{1}{x-3} = -3$$

$$x - 3 = -\frac{1}{3}$$

$$x = 3 - \frac{1}{3} = \frac{8}{3}$$

The x-intercept is $(2.67, 0)$.

$$x = 0$$

$$g(0) = -\frac{1}{3} + 3 = \frac{8}{3}$$

The y-intercept is $(0, 2.67)$.

d $g(x) = \dfrac{1}{x-3} + 3 = \dfrac{1+3(x-3)}{x-3}$

$$= \frac{1+3x-9}{x-3} = \frac{3x-8}{x-3}$$

$$\Rightarrow a = 3, b = -8, c = 1, d = -3$$

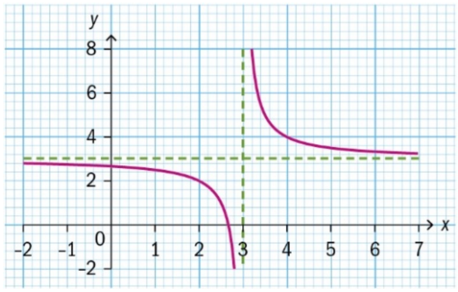
The vertical asymptote is at

$$x = -\frac{d}{c} = -\frac{-3}{1} = 3.$$

The horizontal asymptote is at

$$y = \frac{a}{c} = \frac{3}{1} = 3.$$

e



13 a $f(x) = 2x + 3$

$$x = 2y + 3$$

$$2y = x - 3$$

$$y = \frac{x-3}{2}$$

$$f^{-1}(x) = \frac{x-3}{2}$$

b $g \text{ o} f^{-1}(x) = g\left(\dfrac{x-3}{2}\right) = \dfrac{5}{4\dfrac{x-3}{2}}$

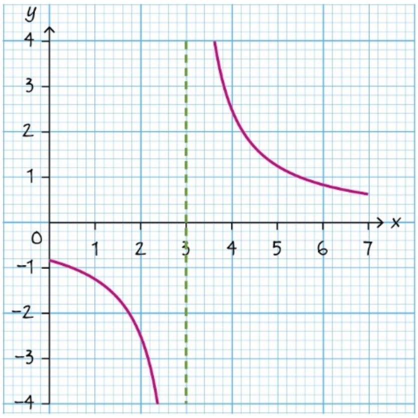
$$= \frac{5}{2(x-3)} = \frac{5}{2x-6}$$

c $x = 0 \Rightarrow h(0) = \dfrac{5}{2 \cdot 0 - 6} = -\dfrac{5}{6}$

The y-intercept of h is

$(0,-\frac{5}{6})=(0,-0.833)$.

d



e $h(x)=\dfrac{5}{2x-6}$

$x=\dfrac{5}{2y-6}$

$x(2y-6)=5$

$2xy-6x=5$

$y=\dfrac{5+6x}{2x}$

$h^{-1}(x)=\dfrac{5+6x}{2x}$

The x-intercept of h^{-1} is given by

$h^{-1}(x)=0$

$\dfrac{5+6x}{2x}=0$

$5+6x=0$

$x=-\dfrac{5}{6}$

The point is therefore

$(-\frac{5}{6},0)=(-0.833,0)$.

f $a=6, b=5, c=2, d=0$

The vertical asymptote is at

$x=-\dfrac{d}{c}=-\dfrac{0}{2}=0$.

14 $f(x)=2+\dfrac{10}{x-4}=\dfrac{2(x-4)+10}{x-4}=\dfrac{2x+2}{x-4}$

$a=2, b=2, c=1, d=-4$

a The vertical asymptote is at

$x=-\dfrac{d}{c}=-\dfrac{-4}{1}=4$.

The horizontal asymptote is at

$y=\dfrac{a}{c}=\dfrac{2}{1}=2$.

b The domain is $x\in\mathbb{R}, x-4\neq 0 \Leftrightarrow x\neq 4$.

The range is $y\in\mathbb{R}, \mathbb{R}-\{2\}$.

c The x-intercept:

$f(x)=0$

$\dfrac{2x+2}{x-4}=0$

$2x+2=0$

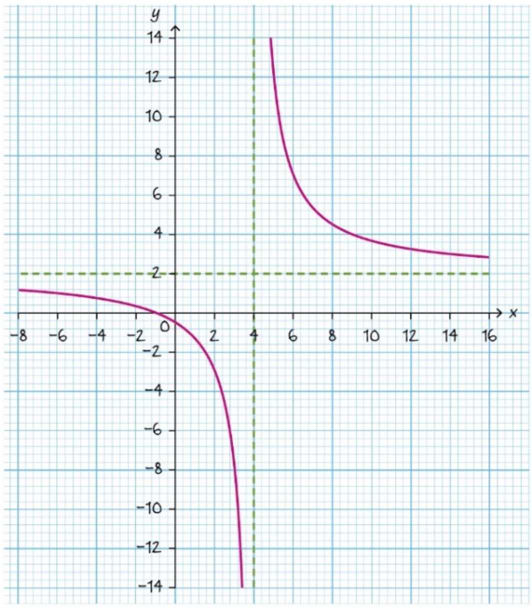
$x=-1$

The point $(-1,0)$.

The y-intercept: $f(0)=\dfrac{2}{-4}=-0.5$

The point $(0,-0.5)$.

d



e Horizontal shift of 4 units right and a vertical shift of 2 units up.

15 a $x\in\mathbb{R}, x\neq -2$ A1

b $f(x)\in\mathbb{R}, f(x)\neq\dfrac{3}{2}$ A1

c When $x=0$, $f(x)=-\dfrac{20}{4}=-5$.

So one coordinate is $(0.-5)$ A1

When $y=0$, $x=\dfrac{20}{3}$

So the other coordinate is $\left(\dfrac{20}{3},0\right)$

A1

16 a Domain is $x\in\mathbb{R}, x\neq -2$

Range is $f(x)\in\mathbb{R}, f(x)\neq 0$ A1A1

b Domain is $x\in\mathbb{R}, x\neq -2$

Range is $f(x)\in\mathbb{R}, f(x)\neq 4$ A1A1

c Domain is $x\in\mathbb{R}, x\neq 0$

Range is $f(x)\in\mathbb{R}, f(x)\neq 4$ A1A1

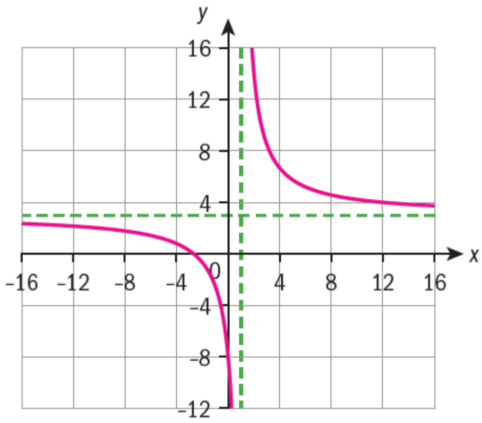
d Domain is $x \in \mathbb{R},\ x \neq 0$

Range is $f(x) \in \mathbb{R},\ f(x) \neq 0$ A1A1

17 a $x = 1$ A1

b $y = 3$ A1

c



A3

18 a $y = 10$ A1

b $x = 2$ A1

c $f(x) = 10 + \dfrac{3}{2-x} = \dfrac{10(2-x)+3}{2-x}$ M1A1

$= \dfrac{-10x+23}{-x+2}$ A1

19 a Vertical asymptote occurs when

$c + 8x = 0$ M1

$c + 8\left(-\tfrac{3}{4}\right) = 0$

$c = 6$ A1

b $y = \dfrac{a+bx}{6+8x}$

Substituting the first coordinate: M1

$\dfrac{2}{5} = \dfrac{a+\frac{1}{2}b}{10}$

$4 = a + \tfrac{1}{2}b$

$8 = 2a + b \quad (1)$ A1

Substituting the second coordinate:

$-\dfrac{3}{38} = \dfrac{a+4b}{38}$

$-3 = a + 4b\ (2)$ A1

Solving **(1)** and **(2)** simultaneously:

$a = 5$ A1

$b = -2$ A1

20 a 6 A1

b $P = \dfrac{18(1+0.82\times 12)}{3+(0.034\times 12)} \approx 57$ M1A1

c Solving $100 = \dfrac{18(1+0.82t)}{3+0.034t}$ M1

$300 + 3.4t = 18(1+0.82t)$

$300 + 3.4t = 18 + 14.76t$

$282 = 11.36t$

$t = \dfrac{282}{11.36} = 24.8$ months A1

d A horizontal asymptote exists at

$P = \dfrac{18\times 0.82}{0.034} = 434.12$ M1A1

Therefore for $t \geq 0$, $P < 435$ R1

21 a $f(x) = \dfrac{17-10x}{2x-1} = \dfrac{12+5-10x}{2x-1}$ M1A1

$= \dfrac{12+5(1-2x)}{2x-1}$ A1

$= \dfrac{12-5(2x-1)}{2x-1}$

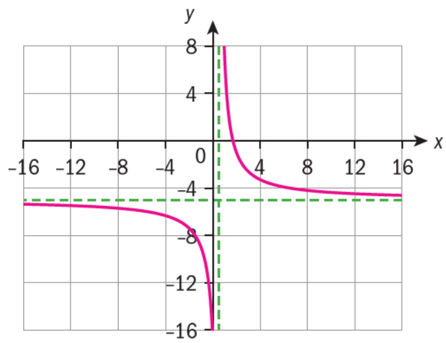
$= \dfrac{12}{2x-1} - \dfrac{5(2x-1)}{(2x-1)}$

$= \dfrac{12}{2x-1} - 5$ A1

b $x = \dfrac{1}{2}$ A1

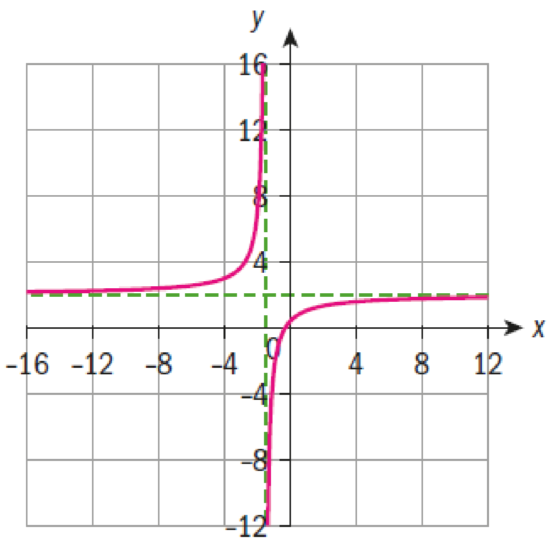
c $y = -5$ A1

d



A3

22



A2

Asymptotes are $x = -\frac{3}{2}$ and $y = 2$

A1A1

Intersections with axes are at $\left(0, \frac{1}{3}\right)$ and $\left(-\frac{1}{4}, 0\right)$

A1A1