

$$23.3 \text{ a) } \sqrt{2x-1} = 2x-3 \quad | ()^2$$

$$\Leftrightarrow (\sqrt{2x-1})^2 = (2x-3)^2$$

$$\Leftrightarrow 2x-1 = (2x)^2 - 2 \cdot 2x \cdot 3 + 3^2$$

$$\Leftrightarrow 2x-1 = 4x^2 - 12x + 9$$

$$\Leftrightarrow 0 = 4x^2 - 14x + 10 \quad \Leftrightarrow x = \begin{cases} \frac{5}{2} \\ 1 \end{cases}$$

$$\text{Test. } x = \frac{5}{2}: \sqrt{2 \cdot \frac{5}{2} - 1} = 2 \cdot \frac{5}{2} - 3 \quad \Leftrightarrow \sqrt{4} = 5 - 3 \quad \checkmark$$

$$x = 1: \sqrt{2 \cdot 1 - 1} = 2 \cdot 1 - 3 \quad \Leftrightarrow \sqrt{1} = -1 \quad \downarrow$$

$$\underline{\text{Varf. } x = \frac{5}{2}}$$

$$\text{b) } \sqrt{2-x} = x+4 \quad | ()^2$$

$$\Leftrightarrow (\sqrt{2-x})^2 = (x+4)^2$$

$$\Leftrightarrow 2-x = x^2 + 2 \cdot x \cdot 4 + 4^2$$

$$\Leftrightarrow 0 = x^2 + 9x + 14 \quad \Leftrightarrow x = \begin{cases} -2 \\ -7 \end{cases}$$

$$\text{Test. } x = -2: \sqrt{2 - (-2)} = -2 + 4 \quad \Leftrightarrow \sqrt{4} = 2 \quad \checkmark$$

$$x = -7: \sqrt{2 - (-7)} = -7 + 4 \quad \Leftrightarrow \sqrt{9} = -3 \quad \downarrow \quad \underline{\text{Varf. } x = -2}$$

$$23.4 \text{ a) } \sqrt{x+1} - 1 = 2x$$

$$\Leftrightarrow \sqrt{x+1} = 2x+1 \quad | ()^2$$

$$\Leftrightarrow (\sqrt{x+1})^2 = (2x+1)^2$$

$$\Leftrightarrow x+1 = (2x)^2 + 2 \cdot 2x \cdot 1 + 1^2$$

$$\Leftrightarrow 0 = 4x^2 + 3x \quad \Leftrightarrow x(4x+3) = 0$$

$$\Leftrightarrow x=0 \text{ bzw. } 4x+3=0 \quad \Leftrightarrow x=0 \text{ bzw. } = -\frac{3}{4}$$

$$\text{Test. } x=0: \sqrt{0+1} - 1 = 2 \cdot 0 \quad \Leftrightarrow 0 = 0 \quad \checkmark$$

$$x = -\frac{3}{4}: \sqrt{-\frac{3}{4}+1} - 1 = 2 \cdot (-\frac{3}{4}) \quad \Leftrightarrow \sqrt{\frac{1}{4}} - 1 = -\frac{3}{2}$$

$$\Leftrightarrow \frac{1}{2} - 1 = -\frac{3}{2} \quad \downarrow$$

$$\underline{\text{Varf. } x=0}$$

$$\text{b) } 8x + 2\sqrt{1-x} = 5$$

$$\Leftrightarrow 2\sqrt{1-x} = 5 - 8x \quad | ()^2$$

$$\Leftrightarrow (2\sqrt{1-x})^2 = (5-8x)^2$$

$$\Leftrightarrow 4(1-x) = 5^2 - 2 \cdot 5 \cdot 8x + (8x)^2$$

$$\Leftrightarrow 4 - 4x = 25 - 80x + 64x^2$$

$$\Leftrightarrow 0 = 64x^2 - 76x + 21 \quad \Leftrightarrow x = \begin{cases} \frac{3}{4} \\ \frac{7}{16} \end{cases}$$

$$\text{Test. } x = \frac{3}{4}: 6 + 1 = 5 \quad \downarrow$$

$$x = \frac{7}{16}: \frac{7}{2} + \frac{3}{2} = 5 \quad \checkmark$$

$$\underline{\text{Varf. } x = \frac{7}{16}}$$

$$24.8 \text{ a) } \varphi(x) = \sqrt[3]{x} - 2\sqrt[4]{x} = 0, \quad \underline{x \geq 0} \quad (x \in [0, \infty[)$$

$$\Leftrightarrow \sqrt[3]{x} = 2\sqrt[4]{x} \quad | ()^{12}$$

$$\Leftrightarrow (\sqrt[3]{x})^{12} = (2 \cdot \sqrt[4]{x})^{12}$$