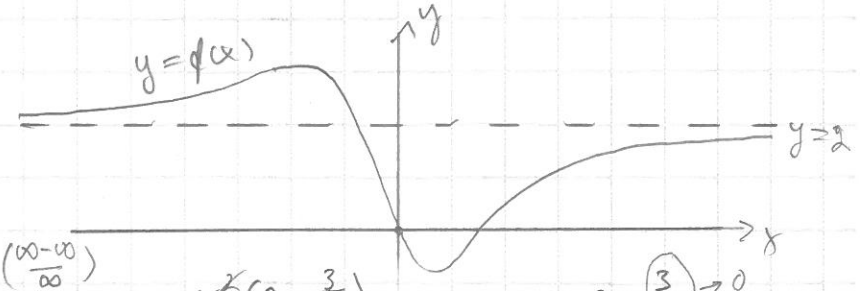
8. Funktion raja-arvo äärettömyydessä

Esim. $f(x) = \dfrac{2x^2 - 3x}{\underbrace{x^2+1}_{\geq 0}}$, $x \in \mathbb{R}$



$$\lim_{x \to \pm\infty} f(x) = \lim_{x \to \pm\infty} \frac{2x^2 - 3x}{x^2+1} \overset{\left(\frac{\infty-\infty}{\infty}\right)}{=} \lim_{x \to \pm\infty} \frac{x^2\left(2 - \frac{3}{x}\right)}{x^2\left(1 + \frac{1}{x^2}\right)} = \lim_{x \to \pm\infty} \frac{2 - \underbrace{\frac{3}{x}}_{\to 0}}{1 + \underbrace{\frac{1}{x^2}}_{\to 0}}$$

$$= \frac{2-0}{1+0} = 2$$

Yleisesti $\lim\limits_{x \to \pm\infty} \dfrac{f(x)}{g(x)}$, f ja g polynomeja

1° g:n korkein potenssi (x^n) yhteiseksi tekijäksi f:stä ja g:stä
2° supistetaan x^n
3° lasketaan raja-arvo ($\lim\limits_{x \to \pm\infty} \frac{1}{x^n} = 0$, $n = 1, 2, \ldots$)

Epämääräisiä muotoja: $\frac{\infty}{\infty}$, $\infty - \infty$, $0 \cdot \infty$

8.2 a) $$\lim_{x \to \infty} \frac{8x^2 - 3x}{4x^2 - 5} \overset{\left(\frac{\infty}{\infty}\right)}{=} \lim_{x \to \infty} \frac{x^2\left(8 - \frac{3}{x}\right)}{x^2\left(4 - \frac{5}{x^2}\right)} = \lim_{x \to \infty} \frac{8 - \frac{3}{x}}{4 - \frac{5}{x^2}} = \frac{8-0}{4-0} = \underline{2}$$

b) $$\lim_{x \to -\infty} \frac{5x^2 + 2x}{x^2} \overset{\left(\frac{\infty}{\infty}\right)}{=} \lim_{x \to -\infty} \frac{x^2\left(5 + \frac{2}{x}\right)}{x^2} = \lim_{x \to -\infty} 5 + \frac{2}{x} = 5 + 0 = \underline{5}$$

8.4 a) $$\lim_{x \to -\infty} (9x - x^5) = \lim_{x \to -\infty} \underbrace{x^5}_{\to -\infty}\left(\underbrace{\frac{9}{x^4}}_{\to 0} - 1\right) = (-\infty)^5 \cdot (0-1) = -\infty \cdot (-1) = \underline{\infty}$$

$\left(-\infty - (-\infty) = -\infty + \infty\right)$

b) $$\lim_{x \to -\infty} \frac{6x+1}{\sqrt{9x^2+7}} \overset{\left(\frac{-\infty}{\infty}\right)}{=} \lim_{x \to -\infty} \frac{6x+1}{\sqrt{x^2\left(9 + \frac{7}{x^2}\right)}} = \lim_{x \to -\infty} \frac{6x+1}{\sqrt{x^2}\sqrt{9 + \frac{7}{x^2}}}$$

$$= \lim_{x \to -\infty} \frac{6x+1}{\underbrace{|x|}_{<0}\sqrt{9 + \frac{7}{x^2}}} = \lim_{x \to -\infty} \frac{x\left(6 + \frac{1}{x}\right)}{-x\sqrt{9 + \frac{7}{x^2}}}$$

$$= \lim_{x \to -\infty} \frac{6 + \underbrace{\frac{1}{x}}_{\to 0}}{-\sqrt{9 + \underbrace{\frac{7}{x^2}}_{\to 0}}} = \frac{6+0}{-\sqrt{9+0}} = \frac{6}{-3} = -2$$

8.6 a) $$\lim_{x \to \infty} \frac{1}{7^x} = \frac{1}{\infty} = 0 \qquad 5^{-3} = \frac{1}{5^3}$$

b) $$\lim_{x \to -\infty} \left(\frac{7}{3}\right)^{2x} = \left(\frac{7}{3}\right)^{-\infty} = \frac{1}{\left(\frac{7}{3}\right)^{\infty}} = \frac{1}{\infty} = 0$$

$\left(\frac{7}{3}\right)^{\infty} \to \infty$, kun $\frac{7}{3} > 1$