f jatk. väl. $[-1,3]$ ja deriv. väl. $]-1,3[$

$f'(x) = \frac{1}{2}(-x^2+2x+3)^{-\frac{1}{2}} \cdot (-2x+2) = \frac{-x+1}{\sqrt{-x^2+2x+3}} = 0 \mid \cdot \sqrt{}$

$\Leftrightarrow -x+1=0 \Leftrightarrow x=1$

	-1		1		3
f'		$+$		$-$	
f		$\nearrow$	max	$\searrow$	

$f'(0) = \frac{\sqrt{3}}{3} > 0$, $f'(2) = -\frac{\sqrt{3}}{3} < 0$

pienin arvo: $f(-1) = f(3) = \underline{0}$

suurin arvo: $f(1) = \sqrt{4} = \underline{2}$

17.9 b) Leikkauskohta: $\begin{cases} y = 1-x \\ y = \sqrt{x+1} \end{cases}$ ryh.

$\Rightarrow 1-x = \sqrt{x+1} \quad |()^2 \quad \Leftrightarrow (1-x)^2 = (\sqrt{x+1})^2$

$\Leftrightarrow 1-2x+x^2 = x+1 \Leftrightarrow x^2-3x=0 \Leftrightarrow x = \begin{cases} 0 \\ 3 \end{cases}$

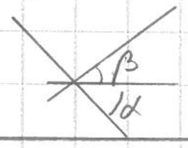
Tark. $x=0$: $1-0 = \sqrt{0+1} \Leftrightarrow 1=1$ ✓.

$x=3$: $1-3 = \sqrt{3+1} \Leftrightarrow -2=2$ ↯

$y = 1-x \Rightarrow y' = -1 = k_{t_1} = \tan\alpha \Rightarrow \alpha = -45^\circ$

$y = \sqrt{x+1} \Rightarrow y' = \frac{1}{2}(x+1)^{-\frac{1}{2}} \Rightarrow k_{t_2} = y'(0) = \frac{1}{2} = \tan\beta$

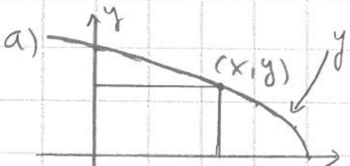
$\Rightarrow \beta = 26{,}57^\circ$



$\Rightarrow$ kulma: $|\beta-\alpha| \approx |26{,}57^\circ-(-45^\circ)|$

$\approx 71{,}57^\circ \approx \underline{71{,}6^\circ}$

17.14 a)



$y = \sqrt{4-x}$ b) Suorakulmion pinta-ala:

$A(x) = xy = x\sqrt{4-x} \underset{x \geq 0}{=} \sqrt{x^2(4-x)}$

$\sqrt{x}$ on aidosti kasvava $\Rightarrow A(x)$

on suurin kun $f(x) = x^2(4-x) = 4x^2-x^3$ on suurin,

f jatk. ja deriv. kun $0 \leq x \leq 4$

$f'(x) = 8x-3x^2 = x(8-3x) = 0 \Leftrightarrow x = \begin{cases} 0 \\ \frac{8}{3} \end{cases}$

	0		$\frac{8}{3}$		4
f'		$+$		$-$	
f		$\nearrow$	max	$\searrow$	

suurin arvo: $A(\frac{8}{3}) = \frac{8}{3}\sqrt{4-\frac{8}{3}} = \underline{\frac{16\sqrt{3}}{9}}$

18.1 a) $D(4\cos x) = -4\sin x$

b) $D(\sin 5x) = \cos 5x \cdot 5 = 5\cos 5x$

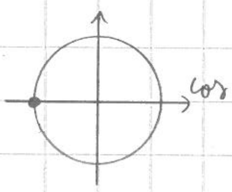
c) $D(\cos x^4) = -\sin x^4 \cdot 4x^3 = -4x^3 \sin x^4$

d) $D(\sin^5 x) = D(\sin x)^5 = 5(\sin x)^4 \cdot \cos x = 5\sin^4 x \cos x$

18.3 $f(x) = x + \sin x$

$f'(x) = 1 + \cos x = 0 \Leftrightarrow \cos x = -1$

$\Leftrightarrow \underline{x = \pi + n2\pi,\ n \in \mathbb{Z}}$



18.5 $f(x) = 4x - \sin 2x$, f jatk. ja deriv. $\mathbb{R}$:ssä

$f'(x) = 4 - \underbrace{\cos 2x}_{-1 \leq \cdot \leq 1} \cdot 2 \geq 4 - 1 \cdot 2 = 2$ aina

$\Rightarrow f$ aidosti kasvava $\mathbb{R}$:ssä

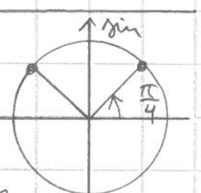
18.7 $f(x) = x\sqrt{2} + \cos 2x$

a) $f'(x) = \sqrt{2} - \sin 2x \cdot 2 = 0$

$\Leftrightarrow \sin 2x = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}} = \sin\frac{\pi}{4}$

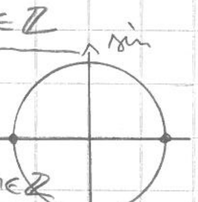
$\Leftrightarrow 2x = \frac{\pi}{4} + n2\pi \mid :2$ tai $2x = \pi - \frac{\pi}{4} + n2\pi = \frac{3\pi}{4} + n2\pi \mid :2$

$\Leftrightarrow \underline{x = \frac{\pi}{8} + n\pi \text{ tai } x = \frac{3\pi}{8} + n\pi,\ n \in \mathbb{Z}}$



b) $f'(x) = \sqrt{2} - \sin 2x \cdot 2 = \sqrt{2}$

$\Leftrightarrow \sin 2x = 0 \Leftrightarrow 2x = n\pi \Leftrightarrow \underline{x = n\frac{\pi}{2},\ n \in \mathbb{Z}}$



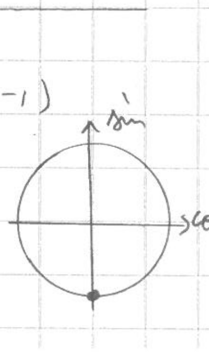
18.8 $f(x) = x^2 + \sin 3x$

$f(\frac{\pi}{2}) = (\frac{\pi}{2})^2 + \sin\frac{3\pi}{2} = \frac{\pi^2}{4} - 1 \Rightarrow$ piste $(\frac{\pi}{2}, \frac{\pi^2}{4}-1)$

$f'(x) = 2x + \cos 3x \cdot 3$

$\Rightarrow k_t = f'(\frac{\pi}{2}) = 2 \cdot \frac{\pi}{2} + \overbrace{\cos\frac{3\pi}{2}}^{=0} \cdot 3 = \pi$

$\Rightarrow$ Tangentti: $y - (\frac{\pi^2}{4}-1) = \pi(x - \frac{\pi}{2}) \Leftrightarrow \underline{y = \pi x - \frac{\pi^2}{4} - 1}$



18.10 $h(t) = 4\sin(\frac{\pi}{6}(t-5)) + 8$ (m)

a) $h(5) = \underline{8\,(m)}$, $h(12) = \underline{6\,(m)}$

b) $h'(t) = 4\cos(\frac{\pi}{6}(t-5)) \cdot \frac{\pi}{6}$ $(\frac{m}{h})$

$h'(5) \approx 2{,}0944 \Rightarrow$ <u>nousee nopeudella 2,1 $\frac{m}{h}$</u>

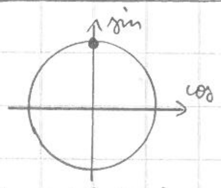
$h'(20) \approx -1{,}8138 \Rightarrow$ <u>laskee — " — 1,8 $\frac{m}{h}$</u>

18.16 $f(x) = A\sin x + B\cos x$

$f'(x) = A\cos x - B\sin x$

$f(\frac{\pi}{2}) = A\sin\frac{\pi}{2} + B\cos\frac{\pi}{2} = A \cdot 1 + B \cdot 0 = \underline{A = \pi}$

$f'(\frac{\pi}{2}) = A\cos\frac{\pi}{2} - B\sin\frac{\pi}{2} = A \cdot 0 - B \cdot 1 = -B = 2 \Leftrightarrow \underline{B = -2}$



18.21 a) $f(x) = \sin x$

$f'(x) = \cos x$, $f''(x) = -\sin x$, $f^{(3)}(x) = -\cos x$, $f^{(4)}(x) = \sin x$

$f^{(5)}(x) = \cos x$, $f^{(6)}(x) = -\sin x$, ...,

$f^{(23)}(x) = f^{(5 \cdot 4 + 3)}(x) = \underline{-\cos x}$

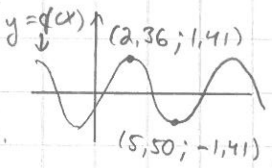
b) $f(x) = \cos x$

$f'(x) = -\sin x$, $f''(x) = -\cos x$, $f^{(3)}(x) = \sin x$, $f^{(4)}(x) = \cos x$

$f^{(5)}(x) = -\sin x$, $f^{(6)}(x) = -\cos x$, ...

$f^{(23)}(x) = f^{(5 \cdot 4 + 3)}(x) = \underline{\sin x}$

19.1 $f(x) = \sin x - \cos x$



f on 2π-jaksollinen funktio $\Rightarrow$ riittää

tutkia väliä $[0, 2\pi]$ missä f jatk. ja deriv.

$f'(x) = \cos x + \sin x = 0$

$\Leftrightarrow \cos x = -\sin x = \sin(-x) = \cos(\frac{\pi}{2} - (-x)) = \cos(\frac{\pi}{2} + x)$

$\Leftrightarrow x = \frac{\pi}{2} + x + n2\pi$ tai $x = -(\frac{\pi}{2} + x) + n2\pi$

$\Leftrightarrow 0 = \frac{\pi}{2} + n2\pi$ ↯ tai $2x = -\frac{\pi}{2} + n2\pi \mid :2$

$\Leftrightarrow x = -\frac{\pi}{4} + n\pi,\ n \in \mathbb{Z} \Rightarrow [0, 2\pi]$: $\frac{3\pi}{4}$ ja $\frac{7\pi}{4}$ ($n=1$ ja $n=2$)

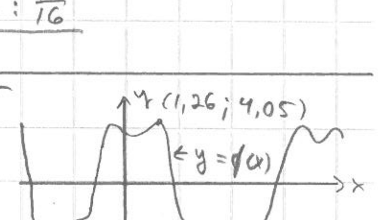
- päätekohdat: $f(0) = f(2\pi) = \sin 0 - \cos 0 = 0 - 1 = -1$

- f':n 0-kohdat: $f(\frac{3\pi}{4}) = \sin\frac{3\pi}{4} - \cos\frac{3\pi}{4} = \frac{\sqrt{2}}{2} - (-\frac{\sqrt{2}}{2}) = \sqrt{2}$

$f(\frac{7\pi}{4}) = \sin\frac{7\pi}{4} - \cos\frac{7\pi}{4} = -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} = -\sqrt{2}$

Vast. <u>suurin arvo: $\sqrt{2}$, pienin arvo: $-\sqrt{2}$</u>

19.3 $f(x) = 4\sin^2 x - 5\sin x + 4$



Mer. $t = \sin x$; $g(t) = 4t^2 - 5t + 4$, $-1 \leq t \leq 1$

g:llä ja f:llä on samat arvojoukot,

g on jatk. ja deriv. väl. $[-1, 1]$

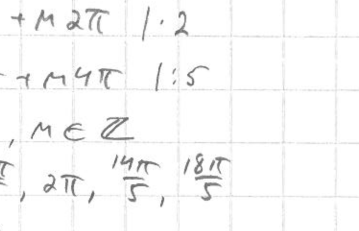
$g'(t) = 8t - 5 = 0 \Leftrightarrow t = \frac{5}{8}$

- päätekohdat: $g(-1) = 13$, $g(1) = 3$

- g':n 0-kohdat: $g(\frac{5}{8}) = \frac{39}{16}$ $(\approx 2{,}438)$

$\Rightarrow$ <u>suurin arvo: 13, pienin arvo: $\frac{19}{16}$</u>

19.5 $f(x) = 4\cos\frac{x}{2} - \cos 2x$



$\begin{cases} \cos\frac{x}{2} \text{ on } \frac{2\pi}{\frac{1}{2}} = 4\pi\text{-jaksollinen} \\ \cos 2x \text{ on } \frac{2\pi}{2} = \pi \text{ — " —} \end{cases}$

$\Rightarrow f$ on 4π-jaksollinen $\Rightarrow$ riittää

tutkia väliä $[0, 4\pi]$ missä f on jatk. ja deriv.

$f'(x) = 4(-\sin\frac{x}{2}) \cdot \frac{1}{2} - (-\sin 2x) \cdot 2 = -2\sin\frac{x}{2} + 2\sin 2x = 0$

$\Leftrightarrow \sin 2x = \sin\frac{x}{2}$

$\Leftrightarrow 2x = \frac{x}{2} + n2\pi \mid \cdot 2$ tai $2x = \pi - \frac{x}{2} + n2\pi \mid \cdot 2$

$\Leftrightarrow 3x = n4\pi \mid :3$ tai $5x = 2\pi + n4\pi \mid :5$

$\Leftrightarrow x = n\frac{4\pi}{3}$ tai $x = \frac{2\pi}{5} + n\frac{4\pi}{5},\ n \in \mathbb{Z}$

$x \in [0, 4\pi]$: $0, \frac{4\pi}{3}, \frac{8\pi}{3}, 4\pi, \frac{2\pi}{5}, \frac{6\pi}{5}, 2\pi, \frac{14\pi}{5}, \frac{18\pi}{5}$

- päätekohdat: $f(0) = f(4\pi) = 3$

- f':n 0-kohdat: $f(\frac{4\pi}{3}) = f(\frac{8\pi}{3}) = -\frac{3}{2}$

$f(\frac{2\pi}{5}) = f(\frac{18\pi}{5}) = \frac{5(\sqrt{5}+1)}{4} \approx 4{,}05$

$f(\frac{6\pi}{5}) = f(\frac{14\pi}{5}) = \frac{-5(\sqrt{5}-1)}{4} \approx -1{,}55$, $f(2\pi) = -5$

<u>suurin arvo: $\frac{5(\sqrt{5}+1)}{4}$, pienin arvo: -5</u>