10. Osamäärän integrointi

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1} + C \quad , n \neq -1$$

$$D \ln|x| = \begin{cases} D \ln x = \frac{1}{x} & , x > 0 \\ D \ln(-x) = \frac{1}{-x} \cdot (-1) = \frac{1}{x} & , x < 0 \end{cases} \Rightarrow \boxed{\int \frac{1}{x} \, dx = \ln|x| + C}$$

$$D \ln|\varphi(x)| = \begin{cases} D \ln \varphi(x) = \frac{1}{\varphi(x)} \cdot \varphi'(x) = \frac{\varphi'(x)}{\varphi(x)} & , \varphi(x) > 0 \\ D \ln(-\varphi(x)) = \frac{1}{-\varphi(x)} \cdot (-\varphi'(x)) = \frac{\varphi'(x)}{\varphi(x)} & , \varphi(x) < 0 \end{cases}$$

$$\Rightarrow \boxed{\int \frac{\varphi'(x)}{\varphi(x)} \, dx = \ln|\varphi(x)| + C}$$

10.7 a) $\int \frac{dx}{2x-6} = \int \frac{1}{2x-6} \, dx = \frac{1}{2} \int \frac{2}{2x-6} \, dx = \frac{1}{2} \ln|\underbrace{2x-6}_{>0 \text{ kun } x>3}| + C$

$$= \underline{\frac{1}{2} \ln(2x-6) + C} \quad , x > 3$$

b) $\int \frac{dx}{(2x-6)^3} = \int \frac{1}{(2x-6)^3} \, dx = \int (2x-6)^{-3} \, dx = \frac{1}{2} \int \underbrace{2}_{\varphi'(x)} \underbrace{(2x-6)^{-3}}_{\varphi(x)} \, dx$

$$= \frac{1}{2} \cdot \frac{1}{-2} (2x-6)^{-2} + C = \underline{-\frac{1}{4(2x-6)^2} + C}$$

c) $\int \frac{dx}{\sqrt{2x-6}} = \int \frac{1}{(2x-6)^{\frac{1}{2}}} \, dx = \frac{1}{2} \int 2 (2x-6)^{-\frac{1}{2}} \, dx = \frac{1}{2} \cdot \frac{1}{\frac{1}{2}} \underbrace{(2x-6)^{\frac{1}{2}}}_{\sqrt{2x-6}} + C$

$$= \underline{\sqrt{2x-6} + C}$$

Huom. jos on laskettava $\int \frac{\varphi(x)}{g(x)} \, dx$

1° Katsotaan saadaanko muotoa $\int \frac{g'(x)}{g(x)} \, dx \to \ln|g(x)| + C$ (10.7 a)

2° Kaytetaan negatiivisia eksponentteja. (10.7 b ja c)

3° Jaetaan termeittain (ks. seuraava esim.)

Esim. $\int \frac{e^{3x} + e^x}{e^x} \, dx = \int \left(\frac{e^{3x}}{e^x} + \frac{e^x}{e^x} \right) dx = \int (e^{2x} + 1) \, dx$

$$= \frac{1}{2} e^{2x} + x + C$$

[TAI: $\int \frac{e^{3x} + e^x}{e^x} \, dx = \int (e^{3x} + e^x) e^{-x} \, dx = \int (e^{2x} + 1) \, dx \ldots$]

10.17 a) $\int \frac{\overbrace{\cos x}^{\varphi'(x)}}{\underbrace{\sin x}_{\varphi(x)}} \, dx = \ln|\underbrace{\sin x}_{>0}| + C = \underline{\ln(\sin x) + C} \quad , 0 < x < \frac{\pi}{2}$

