

$$12.8 \quad a) \log_{12} 3 + 2 \log_{12} 2 = \log_{12} 3 + \log_{12} 2^2 = \log_{12} (3 \cdot 2^2) \\ = \log_{12} 12 = \underline{1}$$

$$b) 5 \log_2 \sqrt[5]{8} = \log_2 (\sqrt[5]{8})^5 = \log_2 8 = \underline{3}$$

$$12.9 \quad \text{Väite} \quad \log \sqrt{ab} = \frac{\log a + \log b}{2}, \quad a, b > 0$$

$$\text{Tod.} \quad \log \sqrt{ab} = \frac{\log a + \log b}{2}$$

$$(\Rightarrow) \log \sqrt{ab} = \frac{\log(ab)}{2} \quad | \cdot 2$$

$$(\Rightarrow) 2 \log \sqrt{ab} = \log(ab)$$

$$(\Rightarrow) \log(\sqrt{ab})^2 = \log(ab)$$

$$(\Rightarrow) \log(ab) = \log(ab) \quad \text{"/, tot} \Rightarrow \text{väite}$$

$$12.15 \quad 1. \log_a x^3 = 3 \log_a x$$

$$2. \log_a \frac{1}{3x} = \log_a 1 - \log_a(3x) = 0 - (\log_a 3 + \log_a x) = -\log_a 3 - \log_a x$$

$$3. -\log_a \frac{3}{x} = -(\log_a 3 - \log_a x) = \log_a x - \log_a 3$$

$$4. \log_a \frac{x}{3} = \log_a x - \log_a 3$$

$$5. -\log_a(3x) = -(\log_a 3 + \log_a x) = -\log_a 3 - \log_a x$$

$$6. -\log_a x - \log_a 3$$

$$7. \log_a x - \log_a 3$$

$$8. 3 \log_a x$$

$$\text{sammutti} = 1, 8 \quad ; \quad 2, 5, 6 \quad ; \quad 3, 4, 7$$

$$12.17 \quad \lg \frac{1}{2} + \lg \frac{2}{3} + \lg \frac{3}{4} + \dots + \lg \frac{98}{99} + \lg \frac{99}{100}$$

$$= (\lg 1 - \lg 2) + (\lg 2 - \lg 3) + (\lg 3 - \lg 4) + \dots + (\lg 98 - \lg 99) + (\lg 99 - \lg 100)$$

$$= \lg 1 - \lg 100 = 0 - 2 = \underline{-2}$$

teleskooppi-ilmiö

$$\text{TAI: } \lg \frac{1}{2} + \lg \frac{2}{3} + \dots + \lg \frac{99}{100} = \lg \left(\frac{1}{2} \cdot \frac{2}{3} \cdot \frac{3}{4} \cdot \dots \cdot \frac{98}{99} \cdot \frac{99}{100} \right) = \lg \frac{1}{100} = -2$$