Deriv. määritelmä 1)

$$\boxed{f'(x_0) = \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0}}$$

Esim. $f(x) = 2x + 1$, $f(-1) = 2\cdot(-1) + 1 = -1$

Tapa 1) $$f'(-1) = \lim_{x \to -1} \frac{f(x) - f(-1)}{x - (-1)}$$

$$= \lim_{x \to -1} \frac{2x + 1 - (-1)}{x + 1}$$

$$= \lim_{x \to -1} \frac{2x + 1 + 1}{x + 1} = \lim_{x \to -1} \frac{2x + 2}{x + 1}$$

$$= \lim_{x \to -1} \frac{2\cancel{(x+1)}}{\cancel{x+1}} = 2$$

Tapn² 2) $f'(x_0) = \lim_{h \to 0} \dfrac{f(x_0+h) - f(x_0)}{h}$

$$f'(-1) = \lim_{h \to 0} \frac{f(-1+h) - f(-1)}{h} = \lim_{h \to 0} \frac{2h - 1 - (-1)}{h}$$

$$= \lim_{h \to 0} \frac{2\cancel{h}}{\cancel{h}} = 2$$

$$f(-1+h) = 2(-1+h) + 1 = 2h - 1$$

Esim.

$$f(x) = \frac{1}{x+1} \qquad f(2) = \frac{1}{2+1} = \frac{1}{3}$$

$$f'(2) = \lim_{x \to 2} \frac{f(x) - f(2)}{x - 2}$$

$$= \lim_{x \to 2} \frac{\frac{1}{x+1}^{(3} - \frac{1}{3}^{(x+1}}{x - 2}$$

$$= \lim_{x \to 2} \frac{\frac{3 - (x+1)}{3(x+1)}}{x - 2}$$

$$= \lim_{x \to 2} \frac{\frac{-x + 2}{3(x+1)}}{x - 2} \quad -x+2 = -(x-2)$$

$$\lim_{x \to 2} \frac{-\cancel{(x-2)}}{3(x+1)} \cdot \frac{1}{\cancel{(x-2)}}$$

$$= \lim_{x \to 2} \frac{-1}{3(x+1)} = -\frac{1}{3(2+1)} = -\frac{1}{9}$$

605. $f(x) = \dfrac{1}{x-3}$, $x \neq 3$

$$\boxed{f'(x_0) = \lim_{x \to x_0} \frac{f(x)-f(x_0)}{x-x_0}}$$

$$f(2) = \frac{1}{2-3} = -\frac{1}{1} = -1$$

$$f'(2) = \lim_{x \to 2} \frac{f(x)-f(2)}{x-2}$$

$(x-3$

$$= \lim_{x \to 2} \frac{\frac{1}{x-3}+1}{x-2}$$

$$= \lim_{x \to 2} \frac{\frac{x-2}{x-3}}{x-2} = \lim_{x \to 2} \frac{\cancel{x-2}}{x-3} \cdot \frac{1}{\cancel{x-2}}$$

$$= \lim_{x \to 2} \frac{1}{x-3} = \frac{1}{2-3} = -1$$

605. jatkuu

$$f'(x_0) = \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0}$$

$$f(x) = \frac{1}{x-3}$$

$$\boxed{f'(x) = \frac{-1}{(x-3)^2}}$$

$$= \lim_{x \to x_0} \frac{{}^{(x_0-3}\dfrac{1}{x-3} - {}^{(x-3}\dfrac{1}{x_0-3}}{x - x_0}$$

$$= \lim_{x \to x_0} \frac{\dfrac{x_0 - 3 - (x-3)}{(x-3)(x_0-3)}}{x - x_0}$$

$$= \lim_{x \to x_0} \frac{\dfrac{x_0 - x}{(x-3(x_0-3)}}{x - x_0}$$

$$x_0 - x = -(x - x_0)$$

$$= \lim_{x \to x_0} \frac{-\cancel{(x - x_0)}}{(x-3)(x_0-3)} \cdot \frac{1}{\cancel{x - x_0}} = \frac{-1}{(x_0-3)(x_0-3)}$$