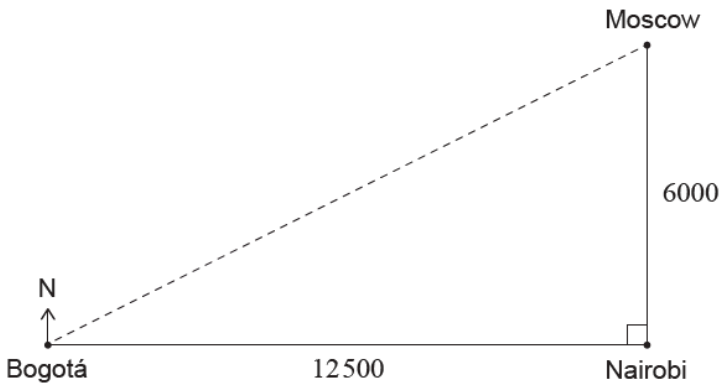
Paper3 /vectors [27 marks]

1. [Maximum mark: 27] 25M.3.AHL.TZ2.2

The following question compares the distance and direction between cities on a flat surface to the distance and direction between cities on a sphere.

Consider a model where the cities of Bogotá, Moscow, and Nairobi lie on a flat surface. In this model, Nairobi is $6000\ \text{km}$ due south of Moscow and Bogotá is $12\,500\ \text{km}$ due west of Nairobi, as shown in the following diagram.

diagram not to scale

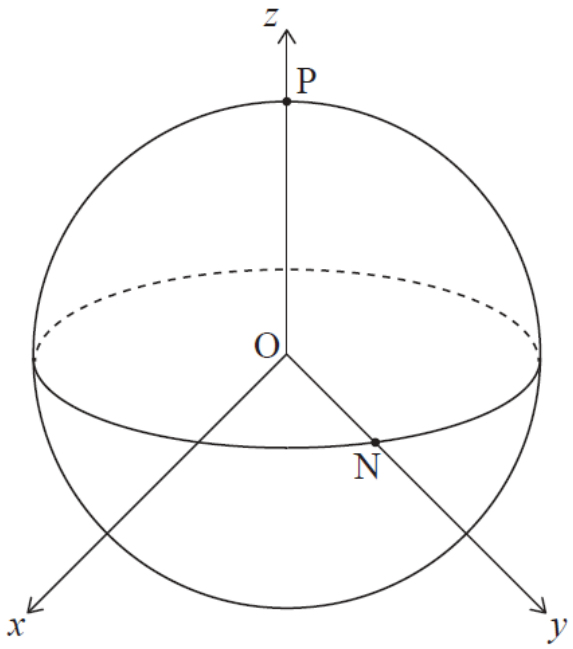


(a.i) Find the distance from Bogotá to Moscow. [2]

(a.ii) Find the bearing of Moscow from Bogotá. Give your answer in degrees. [3]

In reality, these three cities lie on the curved surface of the Earth which will change the distances and directions found in part (a).

Now consider a curved model using a coordinate system $(x,\ y,\ z)$ with its origin, O, at the centre of the Earth. The units of this system are thousands of kilometres and the Earth is modelled as a sphere with radius $6000\ \text{km}$. The North Pole, P, lies on the z-axis, and Nairobi, N, is modelled as being on the equator and lying on the y-axis.

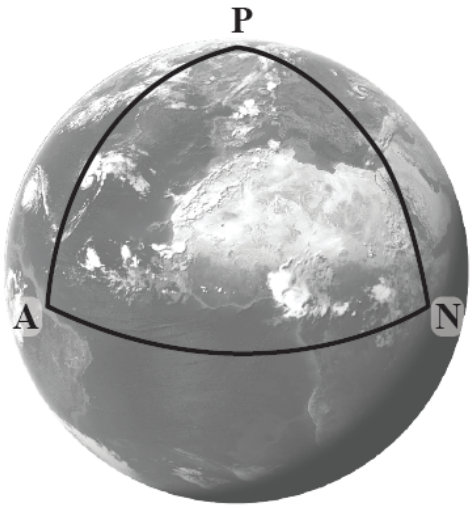


P has position vector $\overrightarrow{\mathrm{OP}} = \boldsymbol{p} = \begin{pmatrix} 0 \\ 0 \\ 6 \end{pmatrix}$ and N has position vector $\overrightarrow{\mathrm{ON}} = \boldsymbol{n} = \begin{pmatrix} 0 \\ 6 \\ 0 \end{pmatrix}$.

(b.i) Use the scalar product to find the angle between $\boldsymbol{p}$ and $\boldsymbol{n}$. [2]

(b.ii) Show that the distance between P and N along the arc from P to N is $3000\pi \text{ km}$. [2]

Point A, which is also on the equator, has position vector $\boldsymbol{a} = \begin{pmatrix} 6 \\ 0 \\ 0 \end{pmatrix}$ as shown in the following diagram.



[Google Maps/Google Earth. Data from Data SIO, NOAA, U.S. Navy, NGA, GEBCOLandsat / Copernicus, IBCAO,
U.S. Geological Survey, PGC/NASA. Imagery from 14/12/2015. Image available at: https://earth.google.com/web/@12.01529518,-18.56070747,-158.39383184a,23597813.93249989d,30.00008083y,359.99981502h,0t,0r/data=CgRCAggBOgMKATBCAggASg0I_____ARAA.]

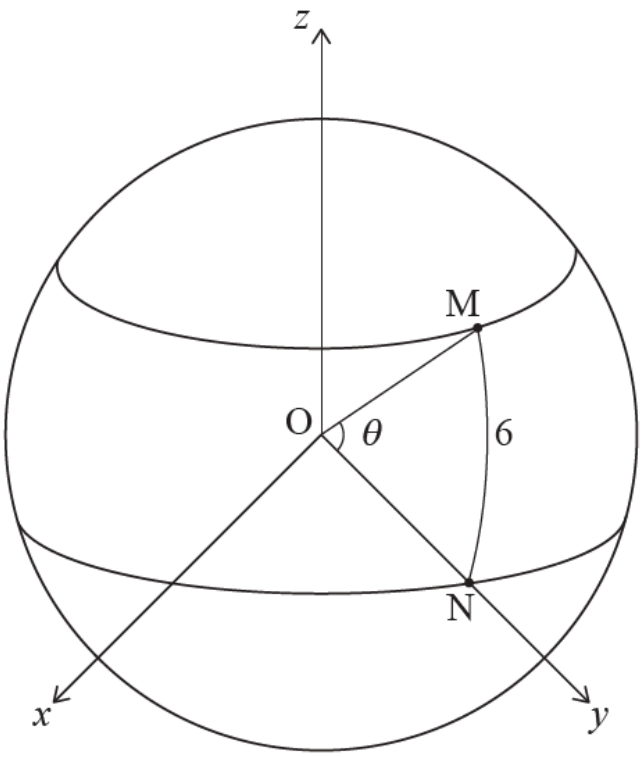
P, N and A, and the arcs connecting them, form a spherical triangle.

The angle at vertex A is defined as the angle between the vectors $\boldsymbol{a} \times \boldsymbol{p}$ and $\boldsymbol{a} \times \boldsymbol{n}$.

(c.i) Find the vector $\boldsymbol{a} \times \boldsymbol{p}$. [2]

(c.ii) Show that the angle at vertex A in the spherical triangle is 90°. [3]

Moscow, M, has position vector $\overrightarrow{\mathrm{OM}} = \boldsymbol{m} = \begin{pmatrix} 0 \\ 6\cos\theta \\ 6\sin\theta \end{pmatrix}$, as shown in the following diagram.



The shortest distance between two points on the sphere lies along an arc of a circle on the sphere with centre O. In this model the shortest distance from Moscow to Nairobi is $6000\ \mathrm{km}$.

(d) Show that $\theta = 57.3^\circ$, correct to three significant figures. [2]

Bogotá, B, is west of Nairobi and has position vector

$$\overrightarrow{\mathrm{OB}} = \boldsymbol{b} = \begin{pmatrix} 6\sin 120^\circ \\ 6\cos 120^\circ \\ 0 \end{pmatrix}.$$

(e) Find the shortest distance from Bogotá to Moscow on the sphere. [5]

The bearing from B to M is defined as the angle at vertex B in the spherical triangle containing B, M and P. It is given that $\boldsymbol{b} \times \boldsymbol{p} = \begin{pmatrix} 36\cos 120^\circ \\ -36\sin 120^\circ \\ 0 \end{pmatrix}$

.

(f) Using the method from part (c), find the bearing from Bogotá to Moscow. [6]

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