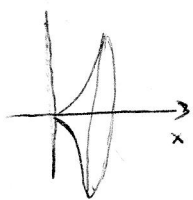


(16.)



$$V = \pi \int_a^b y^2 dx = \pi \int_a^b [f(x)]^2 dx$$

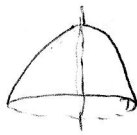
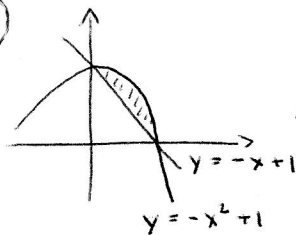
$$y = \sqrt{2x^4 + x^2} \quad [0, \sqrt{2}]$$

$$V = \pi \int_0^{\sqrt{2}} (\sqrt{2x^4 + x^2})^2 dx = \pi \int_0^{\sqrt{2}} (2x^4 + x^2) dx = \pi \int_0^{\sqrt{2}} \left( \frac{2}{5} x^5 + \frac{1}{3} x^3 \right)$$

$$= \pi \left( \frac{2}{5} (\sqrt{2})^5 + \frac{1}{3} (\sqrt{2})^3 - 0 \right) = \pi \left( \frac{2}{5} \sqrt{2} \cdot (\sqrt{2})^4 + \frac{1}{3} \sqrt{2} (\sqrt{2})^2 \right)$$

$$= \pi \left( \frac{8}{5} \sqrt{2} + \frac{2}{3} \sqrt{2} \right) = \pi \left( \frac{24\sqrt{2}}{15} + \frac{10\sqrt{2}}{15} \right) = \pi \left( \frac{34\sqrt{2}}{15} \right) = \frac{34\pi\sqrt{2}}{15}$$

(17.)



Leikkauspisteet:

$$\begin{cases} y = -x + 1 \\ y = -x^2 + 1 \end{cases}$$

$$-x^2 + 1 = -x + 1$$

$$-x^2 + x = 0$$

$$x(-x + 1) = 0$$

$$x = 0 \quad \text{tai} \quad -x + 1 = 0$$

$$x = 1$$

Integroimisrajat:

$$y = 0 + 1 = 1$$

$$y = -1 + 1 = 0$$

$$y = -x^2 + 1$$

$$x^2 = 1 - y$$

$$x = \pm \sqrt{1 - y}$$

$$y = -x + 1$$

$$x = 1 - y$$

$$V = \int_0^1 \pi (\pm \sqrt{1-y})^2 dy - \int_0^1 \pi (1-y)^2 dy$$

$$= \pi \int_0^1 [(1-y) - (1-2y+y^2)] dy = \pi \int_0^1 [-y^2 + y] dy$$

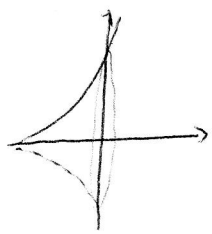
$$= \pi \int_0^1 \left[ -\frac{1}{3} y^3 + \frac{1}{2} y^2 \right] dy = \pi \left[ -\frac{1}{3} \cdot 1^3 + \frac{1}{2} \cdot 1^2 - 0 \right] = \pi \left[ -\frac{2}{6} + \frac{3}{6} \right] = \frac{\pi}{6}$$

(18.)

$$y^2 = (x+4)^3$$

$$\sqrt[3]{y^2} = x+4$$

$$x = \sqrt[3]{y^2} - 4$$

Integroimisrajat:  $y = 0$  ja  $y = 8$ 

$$V = \pi \int_0^8 x^2 dy = \pi \int_0^8 (\sqrt[3]{y^2} - 4)^2 dy$$

$$= \pi \int_0^8 (y^{\frac{2}{3}} - 4)^2 dy = \pi \int_0^8 (y^{\frac{4}{3}} - 8y^{\frac{2}{3}} + 16) dy$$

$$= \pi \int_0^8 \left( \frac{3}{7} y^{\frac{7}{3}} - \frac{24}{5} y^{\frac{5}{3}} + 16y \right)$$

$$= \pi \left[ \frac{3}{7} \cdot 8^{\frac{7}{3}} - \frac{24}{5} \cdot 8^{\frac{5}{3}} + 16 \cdot 8 - 0 \right]$$

$$= \frac{1024}{35} \pi$$