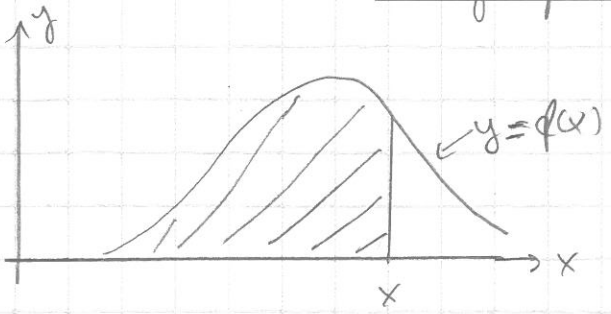
13. Kertymäfunktio



$$F(x) = P(\underline{X} \le x) = \int_{-\infty}^{x} \varphi(t)\,dt$$

KERTYMÄFUNKTIO

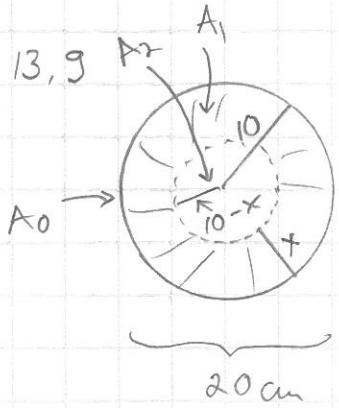
$$\frac{d}{dx}\int_{-\infty}^{x} \varphi(t)\,dt = \frac{d}{dx}\Big/_{a}^{x} G(t) = \frac{d}{dx}\,(G(x) - \overbrace{G(a)}^{\text{vakio}})$$

int.-funktio: $G(t)$ — riittävän jouheva

$$= G'(x) - 0 = \varphi(x)$$

$$\Rightarrow \boxed{F'(x) = \varphi(x)}$$

13,9



$\underline{X}$: tikan etäisyys reunasta (0 ... 10)

$1^\circ\ x \le 0: F(x) = P(\underline{X} \le x) = 0$

$2^\circ\ x \ge 10: F(x) = P(\underline{X} \le x) = 1$

$3^\circ\ 0 < x < 10:$ geometrinen tn

$$F(x) = P(\underline{X} \le x) = \frac{A_1}{A_0} = \frac{A_0 - A_2}{A_0}$$

$$= \frac{A_0}{A_0} - \frac{A_2}{A_0} = 1 - k^2 = 1 - \left(\frac{10-x}{10}\right)^2$$

$$= 1 - \left(1 - \frac{x}{10}\right)^2$$

$$= 1 - \left(1 - \frac{x}{5} + \frac{x^2}{100}\right) = \frac{x}{5} - \frac{x^2}{100}$$

$$\Rightarrow F(x) = \begin{cases} 0, & x \le 0 \\ \frac{x}{5} - \frac{x^2}{100}, & 0 < x < 10 \\ 1, & x \ge 10 \end{cases}$$ kertymäfunktio

$$\Rightarrow \varphi(x) = F'(x) = \begin{cases} \frac{1}{5} - \frac{1}{50}x, & 0 < x < 10 \\ 0, & \text{muualla} \end{cases}$$ tiheysfunktio

$$E(\underline{X}) = \int_{-\infty}^{\infty} x\,\varphi(x)\,dx = \int_{0}^{10} x\left(\frac{1}{5} - \frac{1}{50}x\right)dx = \int_{0}^{10}\left(\frac{1}{5}x - \frac{1}{50}x^2\right)dx$$

$$= \ldots = \frac{10}{3} \approx 3{,}33\ (\text{cm})$$ odotusarvo

$$D\underline{X}: \int_{-\infty}^{\infty} (x - E(\underline{X}))^2 \varphi(x)\,dx = \int_{0}^{10} \underbrace{\left(x - \frac{10}{3}\right)^2} \cdot \left(\frac{1}{5} - \frac{1}{50}x\right)dx$$