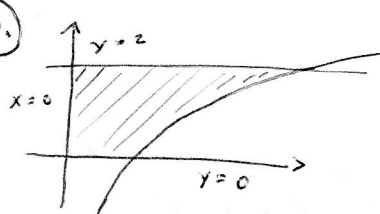


13.



$$y = \ln x$$

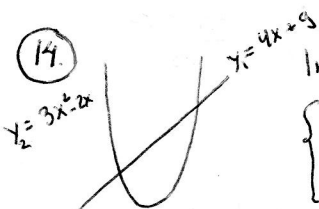
$$x = e^y$$

Integroimisrajat:

$$y = 0 \text{ ja } y = 2$$

$$A = \int_0^2 e^y dy = \left[e^y \right]_0^2 = e^2 - e^0 = e^2 - 1 \approx 6.39$$

14.



Integroimisrajat:

$$\begin{cases} y_1 = 4x + 9 \\ y_2 = 3x^2 - 2x \end{cases}$$

$$y_1 - y_2 > 0, \text{ kun}$$

$$-1 \leq x \leq 3$$

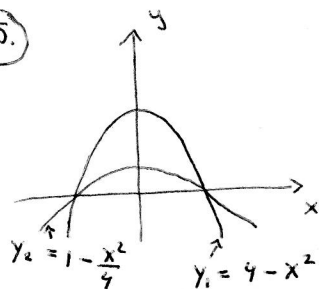
$$3x^2 - 2x = 4x + 9$$

$$3x^2 - 6x - 9 = 0 \quad (\text{CAS-laskin})$$

$$x = -1 \text{ tai } x = 3$$

$$\begin{aligned} A &= \int_{-1}^3 (y_1 - y_2) dx = \int_{-1}^3 (4x + 9 - (3x^2 - 2x)) dx = \int_{-1}^3 (-3x^2 + 6x + 9) dx \\ &= \left[-x^3 + 3x^2 + 9x \right]_{-1}^3 = -3^3 + 3 \cdot 3^2 + 9 \cdot 3 - (-(-1)^3 + 3 \cdot (-1)^2 + 9 \cdot (-1)) \\ &= 32 \end{aligned}$$

15.



Integroimisrajat:

$$\begin{cases} y_1 = 4 - x^2 \\ y_2 = 1 - \frac{x^2}{4} \end{cases}$$

$$y_1 - y_2 > 0, \text{ kun}$$

$$-2 \leq x \leq 2$$

$$4 - x^2 = 1 - \frac{x^2}{4} \quad | \cdot 4$$

$$16 - 4x^2 = 4 - x^2$$

$$3x^2 = 12 \quad | :3$$

$$x^2 = 4$$

$$x = \pm 2$$

$$\begin{aligned} A &= \int_{-2}^2 (y_1 - y_2) dx = \int_{-2}^2 \left(4 - x^2 - \left(1 - \frac{x^2}{4} \right) \right) dx = \int_{-2}^2 \left(3 - \frac{3x^2}{4} \right) dx \\ &= \left[3x - \frac{3x^3}{4} \right]_{-2}^2 = 3 \cdot 2 - \frac{3 \cdot 2^3}{4} - \left(3 \cdot (-2) - \frac{3 \cdot (-2)^3}{4} \right) = 8 \end{aligned}$$