12.4 $$f(x)=\begin{cases}0, & x\le 0\\ \frac{a}{\sqrt[4]{x}}, & 0<x\le 1\\ 0, & x>1\end{cases}$$

a) 1° $f(x)\ge 0$ akkor $\Leftrightarrow \underbrace{\frac{a}{\sqrt[4]{x}}}_{>0}\ge 0 \quad ; 0<x\le 1$

$\Leftrightarrow a\ge 0$

2° $\int_{-\infty}^{\infty} f(x)\,dx = 1 \Leftrightarrow \underbrace{\int_{-\infty}^{0} 0\,dx}_{=0} + \int_0^1 \frac{a}{\sqrt[4]{x}}\,dx + \underbrace{\int_1^{\infty} 0\,dx}_{=0}$

$= a\int_0^1 x^{-\frac{1}{4}}\,dx = a\Big/_0^1 \frac{4}{3}x^{\frac{3}{4}} = a\left(\frac{4}{3}\cdot\underbrace{1^{\frac{3}{4}}}_{=1} - \frac{4}{3}\cdot\underbrace{0^{\frac{3}{4}}}_{=0}\right)$

$= a\cdot\frac{4}{3} = 1 \quad |\cdot\frac{3}{4} \Leftrightarrow a=\frac{3}{4}$

1° és 2° $\Rightarrow \underline{a=\frac{3}{4}}$

b) $E(X)=\int_{-\infty}^{\infty} x f(x)\,dx = \underbrace{\int_{-\infty}^{0} x\cdot 0\,dx}_{=0} + \int_0^1 x\cdot\frac{\frac{3}{4}}{\sqrt[4]{x}}\,dx + \underbrace{\int_1^{\infty} x\cdot 0\,dx}_{=0}$

$= \frac{3}{4}\int_0^1 \underbrace{\frac{x^1}{x^{\frac{1}{4}}}}_{x^{1-\frac{1}{4}}}\,dx = \frac{3}{4}\int_0^1 x^{\frac{3}{4}}\,dx = \frac{3}{4}\Big/_0^1 \frac{4}{7}x^{\frac{7}{4}}$

$= \frac{3}{4}\cdot\frac{4}{7}\left[\underbrace{1^{\frac{7}{4}}}_{=1} - \underbrace{0^{\frac{7}{4}}}_{=0}\right] = \frac{3}{7} \quad (\in[0,1]\%)$

c) $\int_{-\infty}^{\infty}(x-E(X))^2 f(x)\,dx = \int_0^1 (x-\frac{3}{7})^2\cdot\frac{\frac{3}{4}}{\sqrt[4]{x}}\,dx$

$= \int_0^1 \left(x^2-\frac{6}{7}x+\frac{9}{49}\right)x^{-\frac{1}{4}}\,dx$

$= \int_0^1 \left(x^{\frac{7}{4}}-\frac{6}{7}x^{\frac{3}{4}}+\frac{9}{49}x^{-\frac{1}{4}}\right)dx$

$= \Big/_0^1 \frac{4}{11}x^{\frac{11}{4}} - \frac{6}{7}\cdot\frac{4}{7}x^{\frac{7}{4}} + \frac{9}{49}\cdot\frac{4}{3}x^{\frac{3}{4}}$

$= \left(\frac{4}{11}-\frac{6}{7}\cdot\frac{4}{7}+\frac{9}{49}\cdot\frac{4}{3}\right)-0 = \frac{48}{539}$

$D(X)=\sqrt{\frac{48}{539}} = \frac{4\sqrt{33}}{77} \quad (\approx 0{,}30)$