HL / Vectors [22 marks]

1. [Maximum mark: 8] 25M.2.AHL.TZ3.6

Consider the vectors $\boldsymbol{a} = \begin{pmatrix} -5 \\ 7 \end{pmatrix}$, $\boldsymbol{b} = \begin{pmatrix} -8 \\ 9 \end{pmatrix}$ and $\boldsymbol{c} = \begin{pmatrix} p \\ -6 \end{pmatrix}$, where $p \in \mathbb{R}$.

(a) Find an expression, in terms of p, for

(a.i) $\boldsymbol{a} \cdot \boldsymbol{c}$; [2]

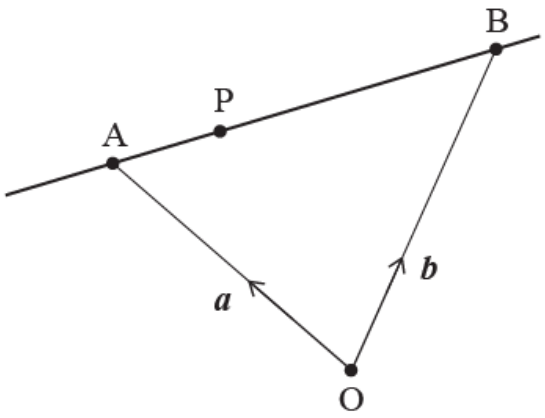
(a.ii) $\boldsymbol{b} \cdot \boldsymbol{c}$. [1]

The angle between $\boldsymbol{a}$ and $\boldsymbol{c}$ is equal to the angle between $\boldsymbol{b}$ and $\boldsymbol{c}$.

(b) Find the value of p. [5]

2. [Maximum mark: 8] 24N.1.AHL.TZ0.8

The following diagram shows two points A and B such that $\overrightarrow{\mathrm{OA}} = \boldsymbol{a}$, $\overrightarrow{\mathrm{OB}} = \boldsymbol{b}$.



The point P lies on (AB) so that $\overrightarrow{\mathrm{AP}} = \lambda \overrightarrow{\mathrm{AB}}$ where $0 < \lambda < 1$.

(a) Show that $\overrightarrow{\mathrm{OP}} = (1-\lambda)\boldsymbol{a} + \lambda\boldsymbol{b}$. [1]

It is given that $|\boldsymbol{a}| = 1$, $|\boldsymbol{b}| = 2$ and $\boldsymbol{a} \cdot \boldsymbol{b} = \frac{1}{4}$.

(b) In the case that $\overrightarrow{\mathrm{OP}}$ is perpendicular to $\overrightarrow{\mathrm{AB}}$, find the value of λ.

[7]

3. [Maximum mark: 6] 22N.2.AHL.TZ0.7

Consider the vectors $\boldsymbol{u} = \boldsymbol{i} + \boldsymbol{j}$ and $\boldsymbol{v} = \left(\cos\frac{1}{n}\right)\boldsymbol{i} + \left(\sin\frac{1}{n}\right)\boldsymbol{j}$, where $n \in \mathbb{Z}^+$.

Let θ be the angle between $\boldsymbol{u}$ and $\boldsymbol{v}$.

(a) Find an expression for $\cos\theta$ in terms of n. [3]

(b) Find the exact value of the limit approached by θ as $n \to \infty$. [3]

© International Baccalaureate Organization, 2026