HL / Binomial theorem *[26 marks]*

1a. Expand and simplify $\left(x-\frac{2}{x}\right)^4$. *[3 marks]*

Markscheme

$\left(x-\frac{2}{x}\right)^4 = x^4 + 4x^3\left(-\frac{2}{x}\right) + 6x^2\left(-\frac{2}{x}\right)^2 + 4x\left(-\frac{2}{x}\right)^3 + \left(-\frac{2}{x}\right)^4$ ***(A2)***

Note: Award ***(A1)*** for 3 or 4 correct terms.

Note: Accept combinatorial expressions, *e.g.* $\binom{4}{2}$ for 6.

$= x^4 - 8x^2 + 24 - \frac{32}{x^2} + \frac{16}{x^4}$ ***A1***

[3 marks]

1b. Hence determine the constant term in the expansion $\left(2x^2+1\right)\left(x-\frac{2}{x}\right)^4$. *[2 marks]*

Markscheme

constant term from expansion of $\left(2x^2+1\right)\left(x-\frac{2}{x}\right)^4 = -64 + 24 = -40$ ***A2***

Note: Award ***A1*** for –64 or 24 seen.

[2 marks]

2. Determine the first three terms in the expansion of $(1-2x)^5(1+x)^7$ in ascending powers of *x*. *[5 marks]*

Markscheme

METHOD 1

constant term: $\binom{5}{0}(-2x)^0\binom{7}{0}x^0 = 1$ ***A1***

term in *x*: $\binom{7}{1}x + \binom{5}{1}(-2x) = -3x$ ***(M1)A1***

term in x^2 : $\binom{7}{2}x^2 + \binom{5}{2}(-2x)^2 + \binom{7}{1}x\binom{5}{1}(-2x) = -9x^2$ ***M1A1***

N3

[5 marks]

METHOD 2

$(1-2x)^5(1+x)^7 = \left(1 + 5(-2x) + \frac{5\times4(-2x)^2}{2!} + \ldots\right)\left(1 + 7x + \frac{7\times6}{2}x^2 + \ldots\right)$

M1M1

$= (1 - 10x + 40x^2 + \ldots)(1 + 7x + 21x^2 + \ldots)$

$= 1 + 7x + 21x^2 - 10x - 70x^2 + 40x^2 + \ldots$

$= 1 - 3x - 9x^2 + \ldots$ ***A1A1A1 N3***

[5 marks]

3. Expand $(2-3x)^5$ in ascending powers of *x*, simplifying coefficients. *[4 marks]*

Markscheme

clear attempt at binomial expansion for exponent 5 ***M1***

$2^5 + 5\times2^4\times(-3x) + \frac{5\times4}{2}\times2^3\times(-3x)^2 + \frac{5\times4\times3}{6}\times2^2\times(-3x)^3$

$+\frac{5\times4\times3\times2}{24}\times2\times(-3x)^4 + (-3x)^5$ ***(A1)***

Note: Only award ***M1*** if binomial coefficients are seen.

$= 32 - 240x + 720x^2 - 1080x^3 + 810x^4 - 243x^5$ ***A2***

Note: Award ***A1*** for correct moduli of coefficients and powers. ***A1*** for correct signs.

Total [4 marks]

4. Find the coefficient of x^{-2} in the expansion of $(x-1)^3\left(\frac{1}{x}+2x\right)^6$. *[6 marks]*

Markscheme

expanding $(x-1)^3 = x^3 - 3x^2 + 3x - 1$ ***A1***

expanding $\left(\frac{1}{2}+2x\right)^6$ gives

$64x^6 + 192x^4 + 240x^2 + \frac{60}{x^2} + \frac{12}{x^4} + \frac{1}{x^6} + 160$ ***(M1)A1A1***

Note: Award ***(M1)*** for an attempt at expanding using binomial.

Award ***A1*** for $\frac{60}{x^2}$.

Award ***A1*** for $\frac{12}{x^4}$.

$\frac{60}{x^2} \times -1 + \frac{12}{x^4} \times -3x^2$ ***(M1)***

Note: Award ***(M1)*** only if both terms are considered.

therefore coefficient x^{-2} is -96 ***A1***

Note: Accept $-96x^{-2}$

Note: Award full marks if working with the required terms only without giving the entire expansion.

[6 marks]

When $\left(1+\frac{x}{2}\right)^n$, $n \in \mathbb{N}$, is expanded in ascending powers of x , the coefficient of x^3 is 70.

5. (a) Find the value of n . *[6 marks]*

(b) Hence, find the coefficient of x^2 .

Markscheme

(a) coefficient of x^3 is $\binom{n}{3}\left(\frac{1}{2}\right)^3 = 70$ ***M1(A1)***

$\frac{n!}{3!(n-3)!} \times \frac{1}{8} = 70$ ***(A1)***

$\Rightarrow \frac{n(n-1)(n-2)}{48} = 70$ ***(M1)***

$n = 16$ ***A1***

(b) $\binom{16}{2}\left(\frac{1}{2}\right)^2 = 30$ ***A1***

[6 marks]