Sequences and series / HL [68 marks]

1. [Maximum mark: 5] 22N.2.SL.TZ0.4

geometric sequence has a first term of 50 and a fourth term of 86.4.

The sum of the first n terms of the sequence is S_n.

Find the smallest value of n such that $S_n > 33\ 500$. [5]

2. [Maximum mark: 6] EXN.1.SL.TZ0.4

The first three terms of an arithmetic sequence are $u_1,\ 5u_1 - 8$ and $3u_1 + 8$.

(a) Show that $u_1 = 4$. [2]

(b) Prove that the sum of the first n terms of this arithmetic sequence is a square number. [4]

3. [Maximum mark: 5] 20N.1.AHL.TZ0.H_5

The first term in an arithmetic sequence is 4 and the fifth term is $\log_2 625$.

Find the common difference of the sequence, expressing your answer in the form $\log_2 p$, where $p \in \mathbb{Q}$. [5]

4. [Maximum mark: 5] 22M.1.SL.TZ2.2

The n^{th} term of an arithmetic sequence is given by $u_n = 15 - 3n$.

(a) State the value of the first term, u_1. [1]

(b) Given that the n^{th} term of this sequence is -33, find the value of n. [2]

(c) Find the common difference, d. [2]

5. [Maximum mark: 5] 21M.2.SL.TZ2.3

An arithmetic sequence has first term 60 and common difference -2.5.

(a) Given that the kth term of the sequence is zero, find the value of k. [2]

(b) Let S_n denote the sum of the first n terms of the sequence.

Find the maximum value of S_n. [3]

6. [Maximum mark: 6] 19N.1.SL.TZ0.S_1

In an arithmetic sequence, $u_2 = 5$ and $u_3 = 11$.

(a) Find the common difference. [2]

(b) Find the first term. [2]

(c) Find the sum of the first 20 terms. [2]

7. [Maximum mark: 7] 19N.2.SL.TZ0.S_5

The first two terms of a geometric sequence are $u_1 = 2.1$ and $u_2 = 2.226$.

(a) Find the value of r. [2]

(b) Find the value of u_{10}. [2]

(c) Find the least value of n such that $S_n > 5543$. [3]

8. [Maximum mark: 7] 18M.2.SL.TZ2.S_4

The first term of an infinite geometric sequence is 4. The sum of the infinite sequence is 200.

(a) Find the common ratio. [2]

(b) Find the sum of the first 8 terms. [2]

(c) Find the least value of n for which $S_n > 163$. [3]

9. [Maximum mark: 5] 17M.1.AHL.TZ2.H_3

The 1st, 4th and 8th terms of an arithmetic sequence, with common difference d, $d \neq 0$, are the first three terms of a geometric sequence, with common ratio r.
Given that the 1st term of both sequences is 9 find

(a) the value of d; [4]

(b) the value of r; [1]

10. [Maximum mark: 6] 17M.1.SL.TZ2.T_5

Tomás is playing with sticks and he forms the first three diagrams of a pattern.
These diagrams are shown below.

Diagram 1 Diagram 2 Diagram 3

Tomás continues forming diagrams following this pattern.

Tomás forms a total of 24 diagrams.

(a) Diagram n is formed with 52 sticks. Find the value of n. [3]

(b) Find the total number of sticks used by Tomás for all 24 diagrams. [3]

11. [Maximum mark: 5] 18N.2.AHL.TZ0.H_1

Consider a geometric sequence with a first term of 4 and a fourth term of −2.916.

(a) Find the common ratio of this sequence. [3]

(b) Find the sum to infinity of this sequence. [2]

12. [Maximum mark: 6] 16N.1.SL.TZ0.T_10

A hydraulic hammer drives a metal post vertically into the ground by striking the top of the post. The distance that the post is driven into the ground, by the $n\text{th}$ strike of the hammer, is d_n.

The distances $d_1,\ d_2,\ d_3\ \ldots,\ d_n$ form a geometric sequence.

The distance that the post is driven into the ground by the first strike of the hammer, d_1, is 64 cm.

The distance that the post is driven into the ground by the second strike of the hammer, d_2, is 48 cm.

(a) Find the value of the common ratio for this sequence. [2]

(b) Find the distance that the post is driven into the ground by the eighth strike of the hammer. [2]

(c) Find the **total depth** that the post has been driven into the ground after 10 strikes of the hammer. [2]