Sequences and series / HL [68 marks]

1. [Maximum mark: 5] 22N.2.SL.TZ0.4

geometric sequence has a first term of 50 and a fourth term of 86.4.

The sum of the first n terms of the sequence is S_n.

Find the smallest value of n such that $S_n > 33\ 500$. [5]

Markscheme

$86.4 = 50r^3$ ***(A1)***

$r = 1.2 \left(= \sqrt[3]{\frac{86.4}{50}} \right)$ seen anywhere ***(A1)***

$\frac{50(1.2^n - 1)}{0.2} > 33500$ OR $250(1.2^n - 1) = 33500$ ***(A1)***

attempt to solve their geometric S_n inequality or equation ***(M1)***

sketch OR $n > 26.9045$, $n = 26.9$ OR $S_{26} = 28368.8$ OR $S_{27} = 34092.6$ OR algebraic manipulation involving logarithms

$n = 27$ accept $n \geq 27$ ***A1***

[5 marks]

2. [Maximum mark: 6] EXN.1.SL.TZ0.4

The first three terms of an arithmetic sequence are $u_1,\ 5u_1 - 8$ and $3u_1 + 8$.

(a) Show that $u_1 = 4$. [2]

Markscheme

* This sample question was produced by experienced DP mathematics senior examiners to aid teachers in preparing for external assessment in the new MAA course. There may be minor differences in formatting compared to formal exam papers.

EITHER

uses $u_2 - u_1 = u_3 - u_2$ **(M1)**

$(5u_1 - 8) - u_1 = (3u_1 + 8) - (5u_1 - 8)$

$6u_1 = 24$ **A1**

OR

uses $u_2 = \frac{u_1 + u_3}{2}$ **(M1)**

$5u_1 - 8 = \frac{u_1 + (3u_1 + 8)}{2}$

$3u_1 = 12$ **A1**

THEN

so $u_1 = 4$ **AG**

[2 marks]

(b) Prove that the sum of the first n terms of this arithmetic sequence is a square number. [4]

Markscheme

$d = 8$ **(A1)**

uses $S_n = \frac{n}{2}(2u_1 + (n-1)d)$ **M1**

$S_n = \frac{n}{2}(8 + 8(n-1))$ **A1**

$= 4n^2$

$= (2n)^2$ **A1**

Note: The final **A1** can be awarded for clearly explaining that $4n^2$ is a square number.

so sum of the first n terms is a square number **AG**

[4 marks]

3. [Maximum mark: 5] 20N.1.AHL.TZ0.H_5

The first term in an arithmetic sequence is 4 and the fifth term is $\log_2 625$.

Find the common difference of the sequence, expressing your answer in the form $\log_2 p$, where $p \in \mathbb{Q}$. [5]

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

$u_5 = 4 + 4d = \log_2 625$ ***(A1)***

$4d = \log_2 625 - 4$

attempt to write an integer (*eg* 4 or 1) in terms of $\log_2$ ***M1***

$4d = \log_2 625 - \log_2 16$

attempt to combine two logs into one ***M1***

$4d = \log_2 \left(\frac{625}{16}\right)$

$d = \frac{1}{4}\log_2 \left(\frac{625}{16}\right)$

attempt to use power rule for logs ***M1***

$d = \log_2 \left(\frac{625}{16}\right)^{\frac{1}{4}}$

$d = \log_2 \left(\frac{5}{2}\right)$ ***A1***

[5 marks]

Note: Award method marks in any order.

4. [Maximum mark: 5] 22M.1.SL.TZ2.2

The n^{th} term of an arithmetic sequence is given by $u_n = 15 - 3n$.

(a) State the value of the first term, u_1. [1]

Markscheme

$u_1 = 12$ ***A1***

[1 mark]

(b) Given that the n^{th} term of this sequence is -33, find the value of n. [2]

Markscheme

$15 - 3n = -33$ ***(A1)***

$n = 16$ ***A1***

[2 marks]

(c) Find the common difference, d. [2]

Markscheme

valid approach to find d ***(M1)***

$u_2 - u_1 = 9 - 12$ OR recognize gradient is -3 OR attempts to solve $-33 = 12 + 15d$

$d = -3$ ***A1***

[2 marks]

5. [Maximum mark: 5] 21M.2.SL.TZ2.3

An arithmetic sequence has first term 60 and common difference -2.5.

(a) Given that the kth term of the sequence is zero, find the value of k. [2]

Markscheme

attempt to use $u_1 + (n-1)d = 0$ ***(M1)***

$60 - 2.5(k-1) = 0$

$k = 25$ ***A1***

[2 marks]

(b) Let S_n denote the sum of the first n terms of the sequence.

Find the maximum value of S_n. [3]

Markscheme

METHOD 1

attempting to express S_n in terms of n ***(M1)***

use of a graph or a table to attempt to find the maximum sum ***(M1)***

$= 750$ ***A1***

METHOD 2

EITHER

recognizing maximum occurs at $n = 25$ ***(M1)***

$S_{25} = \frac{25}{2}(60 + 0),\ S_{25} = \frac{25}{2}(2 \times 60 + 24 \times -2.5)$ ***(A1)***

OR

attempting to calculate S_{24} ***(M1)***

$S_{24} = \frac{24}{2}(2 \times 60 + 23 \times -2.5)$ ***(A1)***

THEN

$= 750$ ***A1***

[3 marks]

6. [Maximum mark: 6] 19N.1.SL.TZ0.S_1

In an arithmetic sequence, $u_2 = 5$ and $u_3 = 11$.

(a) Find the common difference. [2]

Markscheme

valid approach ***(M1)***

eg $11 - 5, 11 = 5 + d$

$d = 6$ ***A1 N2***

[2 marks]

(b) Find the first term. [2]

Markscheme

valid approach ***(M1)***

eg $u_2 - d$, $5 - 6$, $u_1 + (3 - 1)(6) = 11$

$u_1 = -1$ ***A1 N2***

[2 marks]

(c) Find the sum of the first 20 terms. [2]

Markscheme

correct substitution into sum formula

eg $\frac{20}{2}(2(-1) + 19(6))$, $\frac{20}{2}(-1 + 113)$ ***(A1)***

$S_{20} = 1120$ ***A1 N2***

[2 marks]

7. [Maximum mark: 7] 19N.2.SL.TZ0.S_5

The first two terms of a geometric sequence are $u_1 = 2.1$ and $u_2 = 2.226$.

(a) Find the value of r. [2]

Markscheme

valid approach ***(M1)***

eg $\frac{u_1}{u_2}$, $\frac{2.226}{2.1}$, $2.226 = 2.1r$

$r = 1.06$ (exact) ***A1 N2***

[2 marks]

(b) Find the value of u_{10}. [2]

Markscheme

correct substitution ***(A1)***

eg 2.1×1.06^9

3.54790 ***A1 N2***

$u_{10} = 3.55$

[2 marks]

(c) Find the least value of n such that $S_n > 5543$. [3]

Markscheme

correct substitution into S_n formula ***(A1)***

eg $\frac{2.1(1.06^n-1)}{1.06-1}$, $\frac{2.1(1.06^n-1)}{1.06-1} > 5543$, $2.1\left(1.06^n - 1\right) = 332.58$, sketch of S_n and $y = 5543$

correct inequality for n **or** crossover values ***A1***

eg $n > 87.0316$, $S_{87} = 5532.73$ **and** $S_{88} = 5866.79$

$n = 88$ ***A1 N2***

[3 marks]

8. [Maximum mark: 7] 18M.2.SL.TZ2.S_4

The first term of an infinite geometric sequence is 4. The sum of the infinite sequence is 200.

(a) Find the common ratio. [2]

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

correct substitution into infinite sum ***(A1)***

eg $200 = \frac{4}{1-r}$

$r = 0.98$ (exact) ***A1 N2***

[2 marks]

(b) Find the sum of the first 8 terms. [2]

Markscheme

correct substitution ***(A1)***

$\frac{4\left(1-0.98^8\right)}{1-0.98}$

29.8473

29.8 ***A1 N2***

[2 marks]

(c) Find the least value of n for which $S_n > 163$. [3]

Markscheme

attempt to set up inequality (accept equation) ***(M1)***

eg $\frac{4(1-0.98^n)}{1-0.98} > 163, \ \frac{4(1-0.98^n)}{1-0.98} = 163$

correct inequality for n (accept equation) or crossover values ***(A1)***

eg $n > 83.5234$, $n = 83.5234$, $S_{83} = 162.606$ **and** $S_{84} = 163.354$

$n = 84$ ***A1 N1***

[3 marks]

9. [Maximum mark: 5] 17M.1.AHL.TZ2.H_3

The 1st, 4th and 8th terms of an arithmetic sequence, with common difference d, $d \neq 0$, are the first three terms of a geometric sequence, with common ratio r. Given that the 1st term of both sequences is 9 find

(a) the value of d; [4]

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

EITHER

the first three terms of the geometric sequence are $9, 9r$ and $9r^2$ ***(M1)***

$9 + 3d = 9r (\Rightarrow 3 + d = 3r)$ and $9 + 7d = 9r^2$ ***(A1)***

attempt to solve simultaneously ***(M1)***

$9 + 7d = 9\left(\frac{3+d}{3}\right)^2$

OR

the $1^{\text{st}}, 4^{\text{th}}$ and 8^{th} terms of the arithmetic sequence are

$9,\ 9 + 3d,\ 9 + 7d$ ***(M1)***

$\frac{9+7d}{9+3d} = \frac{9+3d}{9}$ ***(A1)***

attempt to solve ***(M1)***

THEN

$d = 1$ ***A1***

[4 marks]

(b) the value of r; [1]

Markscheme

$r = \frac{4}{3}$ ***A1***

Note: Accept answers where a candidate obtains d by finding r first. The first two marks in either method for part (a) are awarded for the same ideas and the third mark is awarded for attempting to solve an equation in r.

[1 mark]

10. [Maximum mark: 6] 17M.1.SL.TZ2.T_5

Tomás is playing with sticks and he forms the first three diagrams of a pattern.
These diagrams are shown below.

Diagram 1 Diagram 2 Diagram 3

Tomás continues forming diagrams following this pattern.

Tomás forms a total of 24 diagrams.

(a) Diagram n is formed with 52 sticks. Find the value of n. [3]

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

$4 + 3(n - 1) = 52$ ***(M1)(A1)***

Note: Award ***(M1)*** for substitution into the formula of the nth term of an arithmetic sequence, ***(A1)*** for correct substitution.

$n = 17$ ***(A1)*** ***(C3)***

[3 marks]

(b) Find the total number of sticks used by Tomás for all 24 diagrams. [3]

Markscheme

$\frac{24}{2}(2 \times 4 + 23 \times 3)$ **OR** $\frac{24}{2}(4 + 73)$ ***(M1)(A1)*****(ft)**

Notes: Award ***(M1)*** for substitution into the sum of the first n terms of an arithmetic sequence formula, ***(A1)*****(ft)** for their correct substitution, consistent with part (a).

924 ***(A1)*****(ft)** ***(C3)***

Note: Follow through from part (a).

[3 marks]

11. [Maximum mark: 5] 18N.2.AHL.TZ0.H_1

Consider a geometric sequence with a first term of 4 and a fourth term of −2.916.

(a) Find the common ratio of this sequence. [3]

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

$u_4 = u_1 r^3 \Rightarrow -2.916 = 4r^3$ ***(A1)***

solving, $r = -0.9$ ***(M1)A1***

[3 marks]

(b) Find the sum to infinity of this sequence. [2]

Markscheme

$S_\infty = \frac{4}{1-(-9)}$ ***(M1)***

$= \frac{40}{19} (= 2.11)$ ***A1***

[2 marks]

12. [Maximum mark: 6] 16N.1.SL.TZ0.T_10

A hydraulic hammer drives a metal post vertically into the ground by striking the top of the post. The distance that the post is driven into the ground, by the $n\text{th}$ strike of the hammer, is d_n.

The distances $d_1,\ d_2,\ d_3\ \ldots,\ d_n$ form a geometric sequence.

The distance that the post is driven into the ground by the first strike of the hammer, d_1, is 64 cm.

The distance that the post is driven into the ground by the second strike of the hammer, d_2, is 48 cm.

(a) Find the value of the common ratio for this sequence. [2]

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

$48 = 64r$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into geometric sequence formula.

$= 0.75\ \left(\frac{3}{4},\ \frac{48}{64}\right)$ ***(A1)*** ***(C2)***

[2 marks]

(b) Find the distance that the post is driven into the ground by the eighth strike of the hammer. [2]

Markscheme

$64 \times (0.75)^7$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into geometric sequence formula or list of eight values using their r. Follow through from part (a), only if answer is positive.

$= 8.54\ (\text{cm})\ (8.54296\ldots\ \text{cm})$ ***(A1)*****(ft)** ***(C2)***

[2 marks]

(c) Find the **total depth** that the post has been driven into the ground after 10 strikes of the hammer. [2]

Markscheme

$\text{depth} = \frac{64\left(1-(0.75)^{10}\right)}{1-0.75}$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into geometric series formula. Follow through from part (a), only if answer is positive.

$= 242(\text{cm})\ (241.583\ldots)$ ***(A1)*****(ft)** ***(C2)***

[2 marks]

© International Baccalaureate Organization, 2023