

Osio A

$$\begin{aligned}
 1) a) \quad & |4-5|^3 - 2^0 - 4 \cdot (3-4)^2 - 5 \cdot 0 \cdot 2 \\
 & = |-1|^3 - 1 - 4 \cdot (-1)^2 - 0 \\
 & = 1^3 - 1 - 4 \cdot 1 = 1 - 1 - 4 = \underline{\underline{-4}}
 \end{aligned}$$

b)

LUKU	KÄÄNTEISKU	VASTALUKU
-4	$-\frac{1}{4}$	4
$\frac{3}{4}$	$\frac{4}{3}$	$-\frac{3}{4}$

$$\begin{aligned}
 & \left(-\frac{1}{4} + \frac{4}{3}\right) : \left(4 - \left(-\frac{3}{4}\right)\right) = \left(\frac{-3+16}{12}\right) : \left(\frac{16+3}{4}\right) \\
 & = \frac{13}{12} : \frac{19}{4} = \frac{13}{12} \cdot \frac{4}{19} = \frac{13}{57}
 \end{aligned}$$

$$c) \quad \frac{3 - 2 \cdot \frac{1}{3}}{3 + 2 \cdot \frac{1}{3}} = \frac{3 - \frac{2}{3}}{3 + \frac{2}{3}} = \frac{\frac{7-2}{3}}{\frac{9+2}{3}} = \frac{\frac{5}{3}}{\frac{11}{3}} = \frac{5}{11}$$

$$2) a) \quad (a^2 b)^2 \cdot \frac{a b^3}{a^4 b} = a^4 b^2 \cdot \frac{b^2}{a^3} = \underline{\underline{a b^4}}$$

$$b) \quad \frac{2^3 \cdot 8^2}{4^6} = \frac{2^3 \cdot (2^3)^2}{(2^2)^6} = \frac{2^3 \cdot 2^6}{2^{12}} = \frac{2^9}{2^{12}} = \frac{1}{2^3} = \underline{\underline{\frac{1}{8}}}$$

$$c) \quad \sqrt[7]{x^2 \cdot \sqrt[4]{x}} = \sqrt[7]{x^2 \cdot x^{\frac{1}{4}}} = \sqrt[7]{x^{\frac{9}{4}}} = (x^{\frac{9}{4}})^{\frac{1}{7}} = x^{\frac{1}{14}} = \underline{\underline{\sqrt[14]{x}}}$$

$$3) a) \left| \frac{2x-3}{2} - \frac{4-x}{3} = 1 - \frac{5x+1}{6} \right| \cdot 6$$

$$3(2x-3) - 2(4-x) = 6 - (5x+1)$$

$$6x - 9 - 8 + 2x = 6 - 5x - 1$$

$$8x - 17 = +5 - 5x$$

$$13x = 22$$

$$x = \frac{22}{13}$$

$$b) (x^2 - 2x)(4x^2 - 9) = 0$$

$$x^2 - 2x = 0 \quad \text{TA1} \quad 4x^2 - 9 = 0$$

$$x(x-2) = 0 \quad \text{TA1} \quad 4x^2 = 9$$

$$\underline{x=0} \quad \text{TA1} \quad x-2=0 \quad \text{TA1} \quad x^2 = \frac{9}{4}$$

$$\underline{x=2}$$

$$x = \pm \sqrt{\frac{9}{4}} = \pm \frac{3}{2}$$

$$c) \frac{12}{x^2} - \frac{1}{x} - 1 = 0 \quad || \cdot x^2$$

$$12 - x - x^2 = 0$$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4 \cdot (-1) \cdot 12}}{2 \cdot (-1)}$$

$$= \frac{1 \pm \sqrt{1+48}}{-2} = \frac{1 \pm \sqrt{49}}{-2} = \frac{1 \pm 7}{-2}$$

$$x = \frac{8}{-2} = \underline{\underline{-4}}$$

$$\text{TA1} \quad x = \frac{-6}{-2} = \underline{\underline{3}}$$

$$4) a) \quad 3 - x \geq x^2 + 2x - 1$$

$$-x^2 - 3x + 4 \geq 0$$

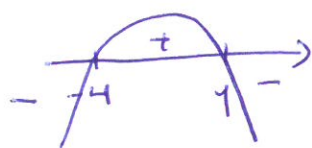
$$-x^2 - 3x + 4 = 0$$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4 \cdot (-1) \cdot 4}}{2 \cdot (-1)} = \frac{3 \pm \sqrt{9+16}}{-2} = \frac{3 \pm 5}{-2}$$

$$x = \frac{8}{-2} = -4$$

TAI

$$x = \frac{-2}{-2} = 1$$



VASTI:

$$x \leq -4$$

TAI

$$x \geq 1$$

$$b) \quad (x-5)(4-2x^2) < 0$$

$$(x-5)(4-2x^2) = 0$$

$$x-5=0$$

TAI

$$4-2x^2=0$$

$$x=5$$

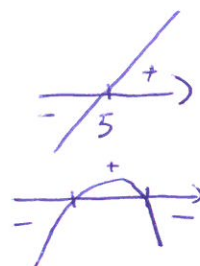
$$-2x^2=-4$$

$$x^2=2$$

$$x = \pm\sqrt{2}$$

MERKKIKAAVIO

	$-\sqrt{2}$	$\sqrt{2}$	5	
$x-5$	-	-	-	+
$4-2x^2$	-	+	-	-
TULO	+	-	+	-



VASTI: $-2 < x < \sqrt{2}$

TAI

$$x > 5$$

$$c) \quad \frac{4-2x}{2x^2-4} \geq 0$$

OSOITAJA

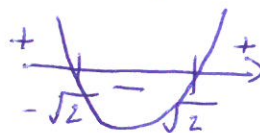
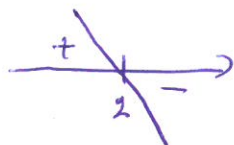
$$\begin{aligned} 4-2x &= 0 \\ -2x &= -4 \\ x &= 2 \end{aligned}$$

NIHITTÄJÄ

$$\begin{aligned} 2x^2-4 &= 0 \\ 2x^2 &= 4 \\ x^2 &= 2 \\ x &= \pm\sqrt{2} \end{aligned}$$

MERKKIKAAVIO

		$-\sqrt{2}$		$\sqrt{2}$		2	
$4-2x$		+		+		+	
$2x^2-4$		+		-		+	
$\frac{4-2x}{2x^2-4}$		+		-		+	



VASTI: $x < -\sqrt{2}$ TAI $\sqrt{2} < x \leq 2$

$$5) a) \quad |18-x| = |4x-12|$$

$$18-x = 4x-12$$

$$-5x = -30$$

$$\underline{x = 6}$$

$$\text{TAI } 18-x = -(4x-12)$$

$$18-x = -4x+12$$

$$3x = -6$$

$$\underline{\underline{x = -2}}$$

MAÄR. EHTO.

$$18-x \geq 0$$

$$-x \geq -18$$

$$x \leq 18$$

$$b) \quad |2x-21| < 3$$

$$-3 < 2x-21 < 3$$

$$18 < 2x < 24$$

$$\underline{\underline{9 < x < 12}}$$

$$c) \begin{cases} 2x - y + 1 = 0 \\ x + 3y - 1 = 0 \end{cases} \begin{array}{l} \cdot (1) \\ \cdot (-2) \end{array}$$

$$\begin{array}{r} 2x - y + 1 = 0 \\ + (-2x - 6y + 2 = 0) \\ \hline \end{array}$$

$$-7y + 3 = 0$$

$$y = \frac{3}{7}$$

JOLLOIN $2x - \frac{3}{7} + 1 = 0$

$$2x = -\frac{4}{7}$$

$$x = -\frac{4}{14} = -\frac{2}{7}$$

VAST: $\begin{cases} x = -\frac{2}{7} \\ y = \frac{3}{7} \end{cases}$

6) a) $(-3, 2)$ JA $(6, -1)$

$$k = \frac{-1-2}{6-(-3)} = \frac{-3}{9} = -\frac{1}{3}$$

$$y - 2 = -\frac{1}{3}(x - (-3))$$

$$y - 2 = -\frac{1}{3}x - 1$$

$$\underline{\underline{y = -\frac{1}{3}x + 1}}$$

b) $(-a, 3)$ JA $(2, a)$

$$k = \frac{a-3}{2-(-a)} = \frac{a-3}{2+a}$$

SUORAN $y = -\frac{5}{2}x - 1$ KULMAKERROIN ON $-\frac{5}{2}$,

JOTEN KOHTISUORASSA OLEVAN SUORAN KULMAKERROIN ON

$$k = \frac{2}{5}$$

NYT

$$\frac{a-3}{2+a} = \frac{2}{5}$$

$$5a - 15 = 4 + 2a$$

$$3a = -19$$

$$a = -\frac{19}{3}$$

$$c) \quad x^2 + y^2 + 2x - 10y + 27 = 0$$

$$x^2 + 2x + 1^2 + y^2 - 10y + 5^2 = -27 + 1^2 + 5^2$$

$$(x+1)^2 + (y-5)^2 = -27 + 1 + 25 = -1$$

VASTI: YHTÄZÖ EI ESITÄ YMPYRÄÄ, KOSKA

SÄDE EI VOI OLLA NEGATIIVINEN

$$7) a) \quad A = (-6, 4, -2)$$

$$\underline{\underline{\vec{OP} = -6\vec{i} + 4\vec{j} - 2\vec{k}}}$$

$$b) \quad \vec{a} = 4\vec{i} + 5\vec{j} - 3\vec{k}$$

$$|\vec{a}| = \sqrt{4^2 + 5^2 + (-3)^2} = \sqrt{16 + 25 + 9} = \sqrt{50} = \sqrt{2 \cdot 25} = \underline{\underline{5\sqrt{2}}}$$

$$c) \quad A = (2, -1, -3)$$

$$B = (0, -1, -2)$$

$$\vec{AB} = (0-2)\vec{i} + (-1-(-1))\vec{j} + (-2-(-3))\vec{k}$$

$$= \underline{\underline{-2\vec{i} + \vec{k}}}$$

$$d) \quad \begin{cases} \vec{a} = 3\vec{i} + 9\vec{j} - 3\vec{k} \\ \vec{b} = 2\vec{i} - 2\vec{j} - 4\vec{k} \end{cases}$$

$$\vec{a} \cdot \vec{b} = 3 \cdot 2 + 9 \cdot (-2) - 3 \cdot (-4)$$

$$= 6 - 18 + 12$$

$$= 18 - 18 = 0$$

VASTI: OVAT KOHTISUORASSA

$$8/a) f(x) = \frac{2x-1}{e^{2x}}$$

$$f'(x) = \frac{2 \cdot e^{2x} - (2x-1) \cdot e^{2x} \cdot 2}{(e^{2x})^2} = \frac{2e^{2x}(1-2x+1)}{e^{4x}} = \frac{2(2-2x)}{e^{2x}}$$

$$= \frac{4-4x}{e^{2x}}$$

$$b) g(x) = 2x \cos 4x$$

$$g'(x) = 2 \cos 4x + 2x (-\sin 4x) \cdot 4$$

$$= 2 \cos 4x - 8x \sin 4x$$

$$c) h(x) = (x^{50} + 1)^{250}$$

$$h'(x) = 250 (x^{50} + 1)^{249} \cdot 50x^{49} = \underline{12500x^{49}(x^{50} + 1)^{249}}$$

$$d) i(x) = \sqrt{x^2 + 1} = (x^2 + 1)^{\frac{1}{2}}$$

$$i'(x) = \frac{1}{2} (x^2 + 1)^{-\frac{1}{2}} \cdot 2x = \underline{\frac{x}{\sqrt{x^2 + 1}}}$$

$$9/a) f(x) = x^3 - 3x^2, \quad x \in [-1, 1]$$

JATKUVAN JA DERIVOITUVAN SUURIN JA PIENIN ARVO
SULJETULLA VÄLILLÄ LÖYTYY VÄLIN PÄÄTEPisteistä TAI
DERIVAATAN NOLLAKOHDISTA

$$f'(x) = 3x^2 - 6x = 0$$

$$x(3x - 6) = 0$$

$$x = 0 \quad \text{TAI} \quad 3x - 6 = 0$$

$$3x = 6$$

$$(x = 2)$$

EI KUULU ANNETULLE VÄLILLE

$$f(-1) = (-1)^3 - 3 \cdot (-1)^2 = -1 - 3 \cdot 1 = -4$$

$$f(1) = 1^3 - 3 \cdot 1 = 1 - 3 = -2$$

$$f(0) = 0^3 - 3 \cdot 0 = 0$$

VASTI: SUURIN ARVO ON 0

PIENIN ARVO ON -4

b) $f(x) = \frac{\ln x}{x}$ MÄÄR. EHTO $x > 0$

FUNKTIO ON JATKOVA JA DERIVOITAVA, KUO $x > 0$

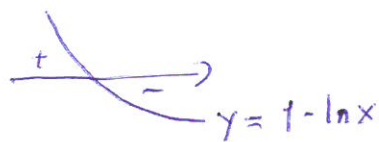
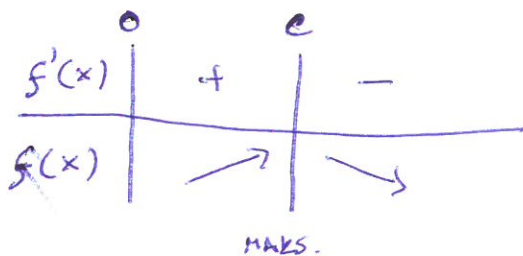
$$f'(x) = \frac{\frac{1}{x} \cdot x - \ln x \cdot 1}{x^2} = \frac{1 - \ln x}{x^2} = 0$$

$$1 - \ln x = 0$$

$$\ln x = 1$$

$$x = e$$

MERKKIKAAVIO



MAKSIMIARVO $f(e) = \frac{\ln e}{e} = \frac{1}{e}$

10) a) OLKOOON KYSYTTY LUKU x

NYT $x = \frac{1}{2} \sqrt{x} \mid \cdot 2$ MÄÄR. EHTO $x \geq 0$

$$2x = \sqrt{x} \mid ()^2$$

$$4x^2 = x$$

$$4x^2 - x = 0$$

$$x(4x - 1) = 0$$

$$x = 0 \quad \text{TAI} \quad 4x - 1 = 0$$

$$4x = 1$$

$$x = \frac{1}{4}$$

VASTI: LUVUT 0 JA $\frac{1}{4}$

$$b) \ln\left(\frac{x}{2}\right) + \ln\left(\frac{e^x}{x}\right) \neq \ln 2$$

$$= \ln x - \ln 2 + \ln e^x - \ln x + \ln 2 = x \cdot \ln e = \underline{x}$$

$$c) \lg(x-1) - \lg 2x = 1$$

MAÄR, EHTO

$$x-1 > 0$$

JÄ

$$2x > 0$$

$$x > 1$$

$$x > 0$$

$$\lg \frac{x-1}{2x} = \lg 10$$

$$\frac{x-1}{2x} = 10$$

$$x-1 = 20x$$

$$-19x = 1$$

$$x = -\frac{1}{19} < 0$$

JOTEU EI TOTEUTA MAÄRITTELYEHTOJA

VAST: EI RATKAISUA

$$11) a) \sin 2x = \cos\left(x + \frac{\pi}{3}\right)$$

$$\cos\left(\frac{\pi}{2} - 2x\right) = \cos\left(x + \frac{\pi}{3}\right)$$

$$\frac{\pi}{2} - 2x = x + \frac{\pi}{3} + n \cdot 2\pi \quad \text{TAI}$$

$$\frac{\pi}{2} - 2x = -x - \frac{\pi}{3} + n \cdot 2\pi$$

$$-3x = \frac{\pi}{3} - \frac{\pi}{2} + n \cdot 2\pi \quad \text{TAI}$$

$$-x = -\frac{\pi}{3} - \frac{\pi}{2} + n \cdot 2\pi$$

$$-3x = -\frac{\pi}{6} + n \cdot 2\pi \quad \text{TAI}$$

$$-x = -\frac{5\pi}{6} + n \cdot 2\pi$$

$$x = \frac{\pi}{18} + n \cdot \frac{2}{3}\pi \quad \text{TAI}$$

$$x = \frac{5\pi}{18} + n \cdot 2\pi$$

b) i) KASVAVA GEOMETRINEN JONO

ESIM. 2, 4, 8, 16, ...

GEOMETRINEN, KOSKA $q = \frac{4}{2} = \frac{8}{4} = \frac{16}{8} = 2$

KASVAVA, KOSKA $4-2 = 2 > 0$

$8-4 = 4 > 0$

ii) VÄHENEVÄ ARITMEETTINEN

ESIM. 6, 5, 4, 3, ...

ARITMEETTINEN, KOSKA $d = 5 - 6 = 4 - 5 = 3 - 4 = -1$

VÄHENEVÄ, KOSKA $5 - 6 = -1 < 0$

$$4 - 5 = -1 < 0$$

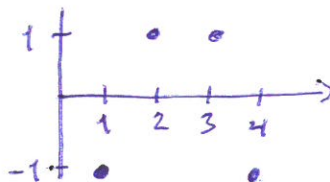
iii) EI OLE ARITMEETTINEN / GEOMETRINEN EIKÄ SUPPENEVA
~~EIKÄ KASVAVA / VÄH~~

ESIM. -1, 1, 1, -1, 1, 1, 1, -1

EI OLE ARITMEETTINEN, KOSKA $1 - (-1) = 2$
 $1 - 1 = 0$ $2 \neq 0$

EI OLE GEOMETRINEN, KOSKA $\frac{1}{-1} = -1$
 $\frac{1}{1} = 1$ $-1 \neq 1$

EI SUPPENE, KOSKA



$$\begin{aligned} 12) a) \int (1+x^2)^2 dx &= \int (1+2x^2+x^4) dx = x + 2 \cdot \frac{1}{3} x^3 + \frac{1}{5} x^5 + C \\ &= \frac{1}{5} x^5 + \frac{2}{3} x^3 + x + C \end{aligned}$$

$$\begin{aligned} b) \int e^{2x} + 2 dx &= \int e^{2x} dx + \int 2 dx \\ &= \frac{1}{2} \int 2e^{2x} dx + \int 2 dx = \frac{1}{2} e^{2x} + 2x + C \end{aligned}$$

$$\begin{aligned} c) \int_0^2 \sqrt[3]{2x} dx &= \int_0^2 (2x)^{1/3} dx = \frac{1}{2} \int_0^2 2(2x)^{1/3} dx \\ &= \frac{1}{2} \cdot \frac{3}{4} (2x)^{4/3} = \frac{3}{8} \left((2 \cdot 2)^{4/3} - (2 \cdot 0)^{4/3} \right) = \frac{3}{8} \cdot 4 \cdot \sqrt[3]{4} = \frac{3}{2} \sqrt[3]{4} \end{aligned}$$