b) $\int \frac{1}{\sqrt{2x+1}}\,dx = \int \frac{1}{(2x+1)^{\frac{1}{2}}}\,dx = \int \underbrace{(2x+1)}_{\varphi(x)}^{-\frac{1}{2}}\,dx$

$= \frac{1}{2}\int \underbrace{2}_{\varphi'(x)}(2x+1)^{-\frac{1}{2}}\,dx = \frac{1}{2}\left(\frac{1}{\frac{1}{2}}(2x+1)^{\frac{1}{2}}\right) + C$

$= \sqrt{2x+1} + C$

8. Exponentialfunktion integrale

$\begin{cases} D\,e^x = e^x \\ D\,e^{\varphi(x)} = e^{\varphi(x)} \cdot \varphi'(x) \end{cases}$

$\int e^x\,dx = e^x + C$

$\int \varphi'(x)\, e^{\varphi(x)}\,dx = e^{\varphi(x)} + C$

Mwon. Si înțelege funcția $\varphi(x)$ derivată față de $\varphi'(x)$ lângă ea în cadrul integralei respective.

Exim. a) $\int e^{3x}\,dx = \frac{1}{3}\int \underbrace{3}_{\varphi'(x)} e^{\underbrace{3x}_{\varphi(x)}}\,dx = \frac{1}{3}\,e^{3x} + C$

b) $\int (3x\,e^{x^2} + e^{-\frac{x}{2}})\,dx = \int 3x\,e^{x^2}\,dx + \int e^{\underbrace{-\frac{x}{2}}_{=-\frac{1}{2}x}}\,dx$

$= 3\cdot\frac{1}{2}\int \underbrace{2x}_{\varphi'(x)} e^{\underbrace{x^2}_{\varphi(x)}}\,dx + (-2)\int \underbrace{-\frac{1}{2}}_{\varphi'(x)} e^{\underbrace{-\frac{x}{2}}_{\varphi(x)}}\,dx$

$= \frac{3}{2}\,e^{x^2} - 2\,e^{-\frac{x}{2}} + C$

c) $\int (e^x + e^{-2x})^2\,dx = \int \left((e^x)^2 + 2\,e^x\,e^{-2x} + (e^{-2x})^2\right)dx$

$= \int (e^{2x} + 2e^{-x} + e^{-4x})\,dx$

$= \frac{1}{2}\int 2\,e^{2x}\,dx + 2(-1)\int -e^{-x}\,dx + \frac{1}{-4}\int -4e^{-4x}\,dx$

$= \frac{1}{2}\,e^{2x} - 2\,e^{-x} - \frac{1}{4}\,e^{-4x} + C$

d) $\int e^{x^2}\,dx$

↖ nu există funcție derivată de $2x$

$\Rightarrow$ nu poate integrată

8.13 Int. rajet: $\varphi(x) = 3 - e^{-x} = 0 \Leftrightarrow 3 = e^{-x} \mid \ln \Leftrightarrow \ln 3 = \ln e^{-x}$

$\Leftrightarrow -x = \ln 3 \Leftrightarrow x = -\ln 3 \ (\approx -1{,}10)$

$A = \int_{-\ln 3}^{0} (3 - e^{-x})\,dx = \Big/_{-\ln 3}^{0} 3x + e^{-x} = (3\cdot 0 + \underbrace{e^0}_{=1}) - (3\cdot(-\ln 3) + e^{-(-\ln 3)})$

$= 1 + 3\ln 3 - 3 = \underline{3\ln 3 - 2} \ (\approx 1{,}30)$

8.19 a) $\int \frac{e^x}{\sqrt{1+2e^x}}\,dx = \int \frac{e^x}{(1+2e^x)^{\frac{1}{2}}}\,dx = \int e^x\,(1+2e^x)^{-\frac{1}{2}}\,dx$

$= \frac{1}{2}\int \underbrace{2e^x}_{\varphi'(x)}\,\underbrace{(1+2e^x)^{-\frac{1}{2}}}_{\varphi(x)}\,dx = \frac{1}{2}\cdot\frac{1}{\frac{1}{2}}(1+2e^x)^{\frac{1}{2}} + C$

$= \sqrt{1+2e^x} + C$