b) $\int \frac{\sin x}{4\cos x}\,dx = \frac{1}{4}\int \frac{\sin x}{\cos x}\,dx = -\frac{1}{4}\int \frac{-\sin x}{\cos x}\,dx$ ($-\sin x = f'(x)$, $\cos x = f(x)$)

$= -\frac{1}{4}\ln|\underbrace{\cos x}_{>0}| + C = \underline{-\frac{1}{4}\ln(\cos x) + C}, \quad 0 < x < \frac{\pi}{2}$

c) $\int \frac{2\cos x}{\sin^2 x}\,dx = \int 2\cos x (\sin x)^{-2}\,dx = 2\int \underbrace{\cos x}_{f'(x)} \underbrace{(\sin x)^{-2}}_{f(x)}\,dx$

$= 2\cdot\frac{1}{-1}(\sin x)^{-1} + C = \underline{-\frac{2}{\sin x} + C}, \quad 0 < x < \frac{\pi}{2}$

10.18 a) $\int_1^e \frac{x-1}{x}\,dx = \int_1^e \left(\underbrace{\frac{x}{x}}_{=1} - \frac{1}{x}\right)dx = \Big/_1^e x - \ln|x|$

$= (e - \underbrace{\ln e}_{=1}) - (1 - \underbrace{\ln 1}_{=0}) = e - 2$

b) $\int_0^1 \underbrace{\frac{e^x}{e^x+2}}_{f'(x)/f(x)}\,dx = \Big/_0^1 \ln(e^x+2) = \ln(e+2) - \ln(\underbrace{e^0}_{=1}+2)$

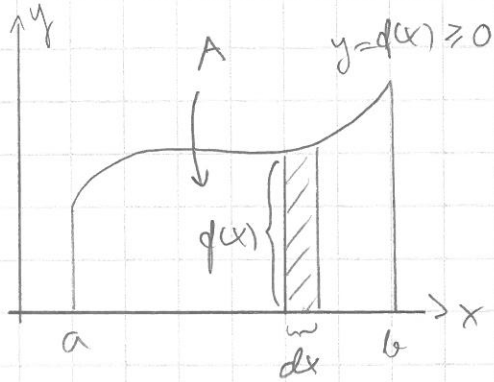
$= \ln(e+2) - \ln 3$

c) $\int_0^1 \frac{e^x+2}{e^x}\,dx = \int_0^1 \frac{e^x}{e^x} + \frac{2}{e^x}\,dx = \int_0^1 (1 + 2e^{-x})\,dx$

$= \Big/_0^1 x - 2e^{-x} = (1 - 2e^{-1}) - (0 - 2\underbrace{e^0}_{=1})$

$= \underline{3 - 2e^{-1}}$

11. Käyrän ja x-akselin välinen alue

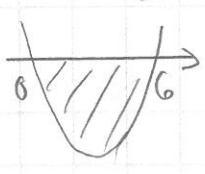


Pylvään:
- kanta: dx
- korkeus: $f(x)$
- pinta-ala: $f(x)\,dx$

Laskemalla pienet pylväät yhteen eli integroimalla: $\boxed{A = \int_a^b f(x)\,dx}$

Huom. Jos $f(x) \le 0$, on $A = -\int_a^b f(x)\,dx$

11.13 $f(x) = x^2 - 6x = 0 \Leftrightarrow x = \begin{cases} 0 \\ 6 \end{cases}$



$A = -\int_0^6 (x^2 - 6x)\,dx = -\Big/_0^6 \frac{1}{3}x^3 - 6\cdot\frac{1}{2}x^2$

$= \ldots = \underline{36}$