6.16 $f(x) = \frac{3}{2}x^4 + 15$, $g(x) = 10x^3 - 12x^2 - 7$

a) $f'(x) = 6x^3 \Rightarrow f'(2) = 6 \cdot 2^3 = 48$

$g'(x) = 30x^2 - 24x \Rightarrow g'(2) = 30 \cdot 2^2 - 24 \cdot 2 = 72$

$g'(2) > f'(2) > 0 \Rightarrow$ g kasvaa jyrkemmin

b) $f'(x) = g'(x)$ (kasvajat yhtä jyrkät)

$\Leftrightarrow 6x^3 = 30x^2 - 24x$

$\Leftrightarrow 6x^3 - 30x^2 + 24x = 0$

$\Leftrightarrow 6x(x^2 - 5x + 4) = 0$

$\Leftrightarrow x = 0$ tai $x^2 - 5x + 4 = 0$

$\Leftrightarrow x = \begin{cases} 1 \\ 4 \end{cases}$

6.19 2. asteen polynomifunktio: $f(x) = ax^2 + bx + c$

$f'(x) = 2ax + b$

$f''(x) = 2a$

f':n derivaattafunktio eli toinen derivaatta

$$\begin{cases} f(2) = a \cdot 2^2 + b \cdot 2 + c = 4a + 2b + c = 4 \\ f'(2) = 2a \cdot 2 + b = 4a + b = -2 \\ f''(2) = 2a = 6 \end{cases}$$

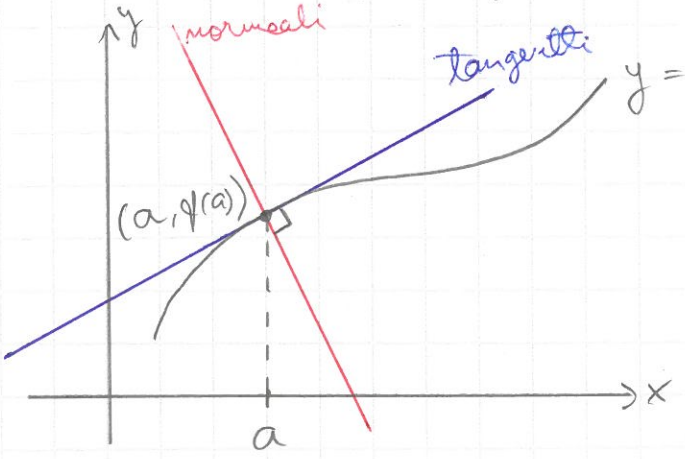
$\Leftrightarrow a = 3$

$\Leftrightarrow b = -2 - 4 \cdot 3 = -14$

$c = 4 - 4 \cdot 3 - 2 \cdot (-14) = 20$

Vast. $f(x) = 3x^2 - 14x + 20$

7. Tangentin ja normaalin yhtälö



「piste: (x_0, y_0), kulmakerroin: k

suora: $y - y_0 = k(x - x_0)$」

$k_t = f'(a)$

$\Rightarrow$ tangentti: $y - f(a) = f'(a)(x - a)$

$k_t \cdot k_n = -1 \Leftrightarrow k_n = -\frac{1}{k_t} = -\frac{1}{f'(a)}$

$\Rightarrow$ normaali: $y - f(a) = -\frac{1}{f'(a)}(x - a)$

7.5 $f(x) = -x^3 + 2x^2 + 1$

$f(2) = -2^3 + 2 \cdot 2^2 + 1 = -8 + 8 + 1 = 1 \Rightarrow$ piste: $(2, 1)$

$f'(x) = -3x^2 + 4x \Rightarrow k_t = f'(2) = -3 \cdot 2^2 + 4 \cdot 2 = -12 + 8 = -4$

$\Rightarrow$ tangentti: $y - 1 = -4(x - 2) \Leftrightarrow y = -4x + 9$