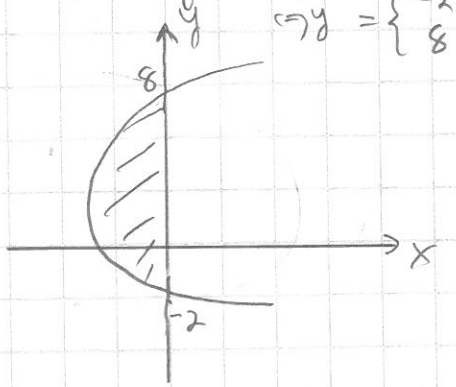
$$\Rightarrow \boxed{A = \int_a^b x\,dy = \int_a^b g(y)\,dy}$$

Hvis for $x = g(y) \le 0$, så $A = -\int_a^b x\,dy = -\int_a^b g(y)\,dy$

12.12 $x = y^2 - 6y - 16 = 0 \Leftrightarrow y = \begin{cases} -2 \\ 8 \end{cases}$

y	x
0	-16
1	-21

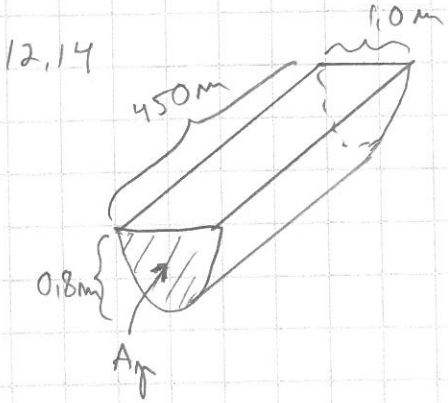


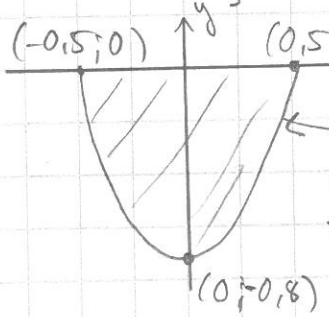
$$A = -\int_{-2}^{8} x\,dy = -\int_{-2}^{8} (y^2 - 6y - 16)\,dy$$

$$= -\Big/_{-2}^{8} \tfrac{1}{3}y^3 - 6\cdot\tfrac{1}{2}y^2 - 16y = -\left[(\dots) - (\dots)\right]$$

$$= \frac{500}{3}$$

12.14





$y = ax^2 + bx + c$

$$\begin{cases} (0,5;0): a\cdot 0,5^2 + b\cdot 0,5 + c = 0 \\ (-0,5;0): a\cdot(-0,5)^2 + b\cdot(-0,5) + c = 0 \\ (0;-0,8): a\cdot 0^2 + b\cdot 0 + c = -0,8 \end{cases}$$

$$\Leftrightarrow \begin{cases} 0,25a + 0,5b + c = 0 & (1) \\ 0,25a - 0,5b + c = 0 & (2) \\ c = -0,8 & (3) \end{cases}$$

(1)+(2): $0,5a + 2c = 0 \Leftrightarrow a = -4c = -4\cdot(-0,8) = 3,2$

(1)−(2): $b = 0$

$\Rightarrow y = 3,2x^2 - 0,8$

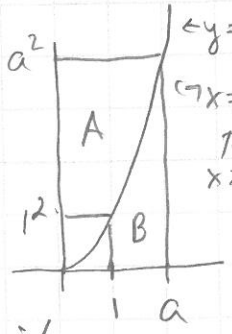
$$A = -\int_{-0,5}^{0,5} (3,2x^2 - 0,8)\,dx = -2\int_0^{0,5} (3,2x^2 - 0,8)\,dx = -2\Big/_0^{0,5} (3,2\cdot\tfrac{1}{3}x^3 - 0,8x)$$

↑ symmetrisk: $y(-x) = y(x) \Rightarrow$ punktet er på parallinen

$$= -2\left[\left(3,2\cdot\tfrac{1}{3}\cdot 0,5^3 - 0,8\cdot 0,5\right) - 0\right] \approx 0,53333\ (m^2)$$

Tilsvarer: $V = A_p h = 0,53333\ m^2 \cdot 450\ m = 240\ m^3$

12.18



$$B = \int_1^a y\,dx = \int_1^a x^2\,dx = \Big/_1^a \tfrac{1}{3}x^3 = \tfrac{1}{3}a^3 - \tfrac{1}{3}\cdot 1^3 = \tfrac{1}{3}a^3 - \tfrac{1}{3}$$

$$A = \int_1^{a^2} x\,dy = \int_1^{a^2} \sqrt{y}\,dy = \int_1^{a^2} y^{\frac{1}{2}}dy = \Big/_1^{a^2} \tfrac{2}{3}y^{\frac{3}{2}}$$

$$= \tfrac{2}{3}(a^2)^{\frac{3}{2}} - \tfrac{2}{3}\cdot 1^{\frac{3}{2}} = \tfrac{2}{3}a^3 - \tfrac{2}{3} = 2\left(\tfrac{1}{3}a^3 - \tfrac{1}{3}\right) = 2B \quad \text{q.e.d.}$$