$F(x) = \int f(x)\,dx = \int ex\,dx = \frac{1}{2}ex^2 + C$

Kulkee pisteen (ln e, e) kautta

$\frac{1}{2}e(\underbrace{\ln e}_{=1})^2 + C = e$

$\frac{1}{2}e + C = e$

$C = e - \frac{1}{2}e = \frac{1}{2}e$

V: $F(x) = \frac{e}{2}x^2 + \frac{1}{2}e$

6. a) $\int x^2(x^3+5)^6\,dx$ $\left\{ u(x) = x^6 \quad s(x) = x^3+5 \quad s'(x) = 3x^2 \right.$

$= \frac{1}{3}\int 3x^2(x^3+5)^6\,dx = \frac{1}{3}\cdot\frac{1}{7}(x^3+5)^7 + C = \frac{1}{21}(x^3+5)^7 + C$

b) $\int \frac{1}{(2x+5)^4}\,dx = \int (2x+5)^{-4}\,dx$ $\left\{ u(x) = x^{-4} \quad s(x) = 2x+5 \quad s'(x) = 2 \right.$

$= \frac{1}{2}\int 2(2x+5)^{-4}\,dx = \frac{1}{2}\cdot\frac{1}{-4+1}(2x+5)^{-3} + C = -\frac{1}{6}(2x+5)^{-3} + C = -\frac{1}{6(2x+5)^3} + C$

c) $\int (xe^{x^2} + e^{-3x})\,dx$ $\left\{ u(x) = e^x \quad s(x) = x^2 \quad s'(x) = 2x \quad s(x) = -3x \quad s'(x) = -3 \right.$

$= \int \left(\frac{1}{2}\cdot 2xe^{x^2} + (-\frac{1}{3})\cdot(-3)e^{-3x}\right)dx = \frac{1}{2}e^{x^2} - \frac{1}{3}e^{-3x} + C$

d) $\int \frac{5+e^{2x}}{e^{2x}}\,dx = \int\left(\frac{5}{e^{2x}} + \frac{e^{2x}}{e^{2x}}\right)dx = \int (5e^{-2x} + 1)\,dx$ $\left\{ u(x) = e^x \quad s(x) = -2x \quad s'(x) = -2 \right.$

$= \int\left((-\frac{1}{2})\cdot 5\cdot(-2)e^{-2x} + 1\right)dx = -\frac{5}{2}e^{-2x} + x + C$

7. a) $\int \frac{\sin x}{\cos^3 x}\,dx = \int \sin x\,(\cos x)^{-3}\,dx$ $\left\{ u(x) = x^{-3}, \quad s(x) = \cos x \quad s'(x) = -\sin x \right.$

$= -\int(-\sin x)(\cos x)^{-3}\,dx = -\left(\frac{1}{-3+1}\right)(\cos x)^{-2} + C = \frac{1}{2\cos^2 x} + C$

b) $\int \frac{x}{3x^2-4}\,dx$ $\left\{ u(x) = \frac{1}{x} \quad s(x) = 3x^2-4 \quad s'(x) = 3\cdot 2x = 6x \right.$

$= \frac{1}{6}\int \frac{6x}{3x^2-4}\,dx = \frac{1}{6}\ln|3x^2-4| + C$

c) $\int \frac{x^4+x^2-3}{x^3}\,dx = \int\left(x + \frac{1}{x} - \frac{3}{x^3}\right)dx = \frac{1}{2}x^2 + \ln|x| - 3\cdot\frac{1}{-3+1}x^{-3+1} + C$

$= \frac{1}{2}x^2 + \ln|x| + \frac{3}{2x^2} + C$

d) $\int \frac{x^2+3x+1}{x+3}\,dx = \int \frac{x(x+3)+1}{x+3}\,dx = \int\left(\frac{x(x+3)}{x+3} + \frac{1}{x+3}\right)dx$

$= \int\left(x + \frac{1}{x+3}\right)dx = \frac{1}{2}x^2 + \ln|x+3| + C$