

3. Murtopotenssi

$$a^m = \underbrace{a \cdot a \cdot \dots \cdot a}_{m \text{ kpl}}$$

$$\boxed{\begin{aligned} a^0 &= 1 \\ a^{-m} &= \frac{1}{a^m} = \left(\frac{1}{a}\right)^m \end{aligned}} \quad a \neq 0$$

Potenssin laskusäännöt:

$$\boxed{\begin{aligned} (ab)^m &= a^m b^m \\ \left(\frac{a}{b}\right)^m &= \frac{a^m}{b^m} \\ a^m a^k &= a^{m+k} \\ \frac{a^m}{a^k} &= a^{m-k} \\ (a^m)^k &= a^{mk} \end{aligned}} \quad \left. \begin{array}{l} \text{ sama eksponentti} \\ \text{ sama kantaluku} \\ \text{ potenssin potenssi} \end{array} \right\}$$

Esim. $(9^{\frac{1}{2}})^2 = 9^{\frac{1}{2} \cdot 2} = 9^1 = 9 \quad \Rightarrow \quad 9^{\frac{1}{2}} = \sqrt{9} = 3$

$(25^{\frac{1}{2}})^2 = 25^{\frac{1}{2} \cdot 2} = 25^1 = 25 \quad \Rightarrow \quad 25^{\frac{1}{2}} = \sqrt{25} = 5$

$(8^{\frac{1}{3}})^3 = 8^{\frac{1}{3} \cdot 3} = 8^1 = 8 \quad \Rightarrow \quad 8^{\frac{1}{3}} = \sqrt[3]{8} = 2$

Yleisesti

$$\boxed{\begin{aligned} a^{\frac{1}{n}} &= \sqrt[n]{a} \\ a^{\frac{k}{n}} &= \sqrt[n]{a^k} = (\sqrt[n]{a})^k \end{aligned}} \quad a > 0$$

potenssin potenssi $\nearrow$ juuren otto $\nwarrow$

Id. $(a^{\frac{1}{n}})^m = a^{\frac{1}{n} \cdot m} = a^1 = a \quad \Rightarrow \quad a^{\frac{1}{n}} = \sqrt[n]{a}$

$$a^{\frac{k}{n}} = \begin{cases} (a^k)^{\frac{1}{n}} = \sqrt[n]{a^k} \\ (a^{\frac{1}{n}})^k = (\sqrt[n]{a})^k \end{cases}$$

9.6 a) $3\sqrt{3} = 3^1 \cdot 3^{\frac{1}{2}} = 3^{1+\frac{1}{2}} = \underline{3^{\frac{3}{2}}}$

b) $9\sqrt{3} = 3^2 \cdot 3^{\frac{1}{2}} = 3^{2+\frac{1}{2}} = \underline{3^{\frac{5}{2}}}$

c) $\frac{1}{27\sqrt{3}} = \frac{1}{3^3 \cdot 3^{\frac{1}{2}}} = \frac{1}{3^{3+\frac{1}{2}}} = \frac{1}{3^{\frac{7}{2}}} = \underline{3^{-\frac{7}{2}}}$

d) $\frac{1}{\sqrt{3^7}} = \frac{1}{3^{\frac{7}{2}}} = \underline{3^{-\frac{7}{2}}}$