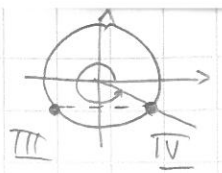


3.18 a) $350^\circ, 710^\circ, -10^\circ$

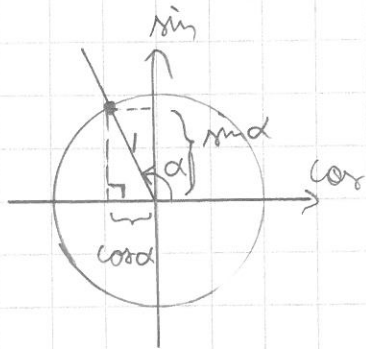
$\uparrow$
 $350^\circ + 360^\circ$

b) $350^\circ, 710^\circ, 190^\circ$
 $\uparrow$
 $180^\circ + 10^\circ$

c) $350^\circ, 710^\circ, -10^\circ$



4. Trigonometrian perusteet



Pythagoras: $\underbrace{(\sin \alpha)^2}_{\text{Merk. } \sin^2 \alpha} + \underbrace{(\cos \alpha)^2}_{\cos^2 \alpha} = 1^2$

$\Rightarrow \boxed{\sin^2 \alpha + \cos^2 \alpha = 1}$

TRIGONOMETRIAN PERUSKAAVA
LUKIOLAISEN ———

$\sin 2\alpha = \sin(\alpha + \alpha) = \sin \alpha \cos \alpha + \cos \alpha \sin \alpha = 2 \sin \alpha \cos \alpha$

$\cos 2\alpha = \cos(\alpha + \alpha) = \cos \alpha \cdot \cos \alpha - \sin \alpha \sin \alpha = \cos^2 \alpha - \sin^2 \alpha$

$= \begin{cases} (1 - \sin^2 \alpha) - \sin^2 \alpha = 1 - 2\sin^2 \alpha \\ \cos^2 \alpha - (1 - \cos^2 \alpha) = 2\cos^2 \alpha - 1 \end{cases}$

perusteet

$\Rightarrow$

$\boxed{\begin{aligned} \sin 2\alpha &= 2 \sin \alpha \cos \alpha \\ \cos 2\alpha &= \cos^2 \alpha - \sin^2 \alpha = 1 - 2\sin^2 \alpha = 2\cos^2 \alpha - 1 \end{aligned}}$

KAKSIKERTAISEN
KUNN SINI JA
KOSINI

- Huom. 1° lukiolaisen perusteet on olettava!
2° 2-kertaisen kulman sinin kaava on hyvä osata.
3° ——— kosini ——— on järkevä kaava.

Esim. $\sin \alpha = \frac{1}{4}$ ja $\frac{\pi}{2} < \alpha < \pi$. Laske $\cos \alpha$, $\sin 2\alpha$ ja $\cos 2\alpha$.

Ratk. Perusteet: $\sin^2 \alpha + \cos^2 \alpha = 1$

$(\Rightarrow) \cos^2 \alpha = 1 - \sin^2 \alpha \quad |\sqrt{}$

$(\Rightarrow) \cos \alpha = \pm \sqrt{1 - \sin^2 \alpha} = -\sqrt{1 - \left(\frac{1}{4}\right)^2} = -\sqrt{1 - \frac{1}{16}} = -\sqrt{\frac{15}{16}} = -\frac{\sqrt{15}}{4}$

$\sin 2\alpha = 2 \sin \alpha \cos \alpha = 2 \cdot \frac{1}{4} \cdot \left(-\frac{\sqrt{15}}{4}\right) = -\frac{\sqrt{15}}{8}$

$\cos 2\alpha = 1 - 2\sin^2 \alpha = 1 - 2 \cdot \left(\frac{1}{4}\right)^2 = 1 - 2 \cdot \frac{1}{16} = 1 - \frac{1}{8} = \frac{7}{8}$

