

Aritmeettinen lukujono

$$a_1 \quad a_2 \quad a_3$$

$$2, 4, 6, 8 \dots$$

$$\begin{array}{c} \curvearrowright \quad \curvearrowright \quad \curvearrowright \\ +2 \quad +2 \quad +2 \end{array}$$

$$(4-2=2)$$

$$(6-4=2)$$

differenssi $d=2$

yleinen jäsen

$$a_n = a_1 + (n-1) \cdot d$$

$$a_{10} = 2 + (10-1) \cdot 2 = 20$$

Summa

$$a_1 \quad a_{10}$$

$$2 + 4 + 6 + \dots + 20$$

$$S_n = n \cdot \frac{a_1 + a_n}{2}$$

$$S_{10} = 10 \cdot \frac{2+20}{2} = 10 \cdot 11 = 110$$

Geometrisen lukujono

$$a_1 \quad a_2 \quad a_3$$

$$2, 4, 8, 16 \dots$$

$$\begin{array}{c} \curvearrowright \quad \curvearrowright \quad \curvearrowright \\ \cdot 2 \quad \cdot 2 \quad \cdot 2 \end{array}$$

suhdeluku $q=2$

$$\left(\frac{4}{2}=2\right)$$

$$\left(\frac{8}{4}=2\right)$$

$$a_n = a_1 \cdot q^{n-1}$$

$$a_{10} = 2 \cdot 2^{10-1} = 1024$$

Summa

$$2 + 4 + 8 \dots + 1024$$

$$S_n = a_1 \cdot \frac{1-q^n}{1-q}$$

$$S_{10} = 2 \cdot \frac{1-2^{10}}{1-2} = 2 \cdot 1023 = 2046$$

A 16.38 Määritä aritmeettisen jonon 15, 18, 21, ...

a) yleinen jäsen

b) 50. jäsen.

$$a_n = 15 + (n-1) \cdot 3$$

$$= 15 + 3n - 3$$

$$= 3n + 12$$

$$b) a_{50} = 3 \cdot 50 + 12$$

$$= 150 + 12$$

$$= 162$$

A 16.39 Aritmeettisen jonon kaksi ensimmäistä jäsentä ovat $a_1 = -24$ ja $a_2 = -18$. Kuinka mones jäsen luku 234 on tässä jonossa?

$$a_1 \quad a_2 \quad a_n$$

$$-24, -18, \dots, 234$$

$$d = -18 - (-24)$$

$$= -18 + 24 = 6$$

$$a_n = -24 + (n-1) \cdot 6$$

$$= -24 + 6n - 6$$

$$= 6n - 30$$

$$6n - 30 = 234 \quad \parallel +30$$

$$6n = 264 \quad \parallel :6$$

$$\underline{n = 44}$$

16.42 Laske aritmeettinen summa

$$124 + 120 + \dots + (-108) + (-112).$$

$$d = 120 - 124 = -4$$

$$\begin{aligned} a_n &= 124 + (n-1) \cdot (-4) \\ &= 124 - 4n + 4 \\ &= -4n + 128 \end{aligned}$$

$$\begin{aligned} -4n + 128 &= -112 \quad // -128 \\ -4n &= -240 \quad // :(-4) \\ n &= 60 \end{aligned}$$

$$S_n = n \cdot \frac{a_1 + a_n}{2}$$

$$S_{60} = 60 \cdot \frac{124 + (-112)}{2} = 60 \cdot 6 = 360$$

16.48 Geometrisen jonon ensimmäinen jäsen on 5 ja suhdeluku 2. Määritä

a) jonon yleinen jäsen

b) a_8 .

$$a_n = 5 \cdot 2^{n-1}$$

$$a_8 = 5 \cdot 2^{8-1} = 5 \cdot 2^7 = 640$$

$$a_n = a_1 \cdot q^{n-1}$$

16.49 Geometrisen jonon yleinen jäsen on

$a_n = 4 \cdot 2^{n-1}$. Laske jonon 10 ensimmäisen jäsenen summa.

$$S_{10} = 4 \cdot \frac{1-2^{10}}{1-2}$$

$$\begin{aligned} &4 \cdot (1-2^{10}) / (1-2) \\ &= 4092 \end{aligned}$$

$$S_n = a_1 \cdot \frac{1-q^n}{1-q}$$

16.51 Laske geometrinen summa

$$1 + 3 + \dots + 1\,594\,323.$$

a_1

$$q = \frac{3}{1} = 3$$

a_n

$$\begin{aligned} a_n &= a_1 \cdot q^{n-1} \\ &= 1 \cdot 3^{n-1} \\ &= 3^{n-1} \end{aligned}$$

$$3^{n-1} = 1\,594\,323$$

$$\begin{aligned} 3^{13} \\ &= 1594323 \end{aligned}$$

saa kokeilla laskimella

$$n-1 = 13$$

$$\begin{aligned} (1-3^{14}) / -2 \\ &= 2391484 \end{aligned}$$

$$n = 14$$

$$S_{14} = 1 \cdot \frac{1-3^{14}}{1-3}$$

$$\begin{aligned} (1-3^{14}) / -2 \\ &= 2391484 \end{aligned}$$

$$\left(\begin{aligned} \text{MAB4:} \\ n-1 &= \log_3 1594323 \end{aligned} \right)$$

$$\begin{aligned} \log(3; 1594323) \\ &= 13 \end{aligned}$$